INFLUENCE OF CHANNEL FLUCTUATIONS ON OPTIMAL REAL-TIME SCALABLE IMAGE TRANSMISSION

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ABSTRACT

Joint source-channel coding systems using scalable source codes and forward error correction allow reliable transmission of multimedia data over noisy channels. The performance of such systems highly depends on the source-channel bit allocation strategy. Rate-based error protection schemes, which maximize the expected source rate are very attractive for real-time applications because the optimization can be done very quickly and is independent of the source. In real-world communication, channel conditions are varying in time. Thus, it is important to frequently update the error protection. For two state-of-the-art joint source-channel coding systems, we show that a channel mismatch can lead to a poor performance. We study theoretically and experimentally the dependency of a rate-based optimal protection on the channel statistics and provide an efficient strategy for adjusting the error protection when a channel mismatch occurs.

1. INTRODUCTION

Scalable image and video coders are very attractive for multimedia communication because they allow progressive decoding and simple rate adaptation. However, the source bitstream is highly sensitive to channel errors, thus a robust error control system is needed. One of the best forward error correction (FEC) based systems for the transmission of scalable bitstreams over a binary symmetric channel (BSC) was proposed by Sherwood and Zeger [1]. The system uses a concatenation of a cyclic redundancy check (CRC) and a rate-compatible punctured convolutional (RCPC) coders to protect the SPIHT [2] code. Error propagation is avoided by stopping the decoding when the first error is detected. For image transmission over packet erasure channels, an efficient FEC-based approach is to partition the source code into layers of decreasing importance and to protect them with increasingly weaker Reed-Solomon codes [3, 4].

If the channel statistics are known, the protection strategy can be optimized by minimizing the expected distortion or by maximizing the expected received source rate [5, 6]. Rate optimization is very desirable for real-time applications because an optimal strategy can be quickly computed [5, 7, 8, 9], and it is independent of both the source coder and the image. Thus, it can be determined by the decoder, which avoids the need for sending side information.

In many popular networks, including the Internet and wireless channels, channel conditions are rapidly varying in time. Therefore a protection scheme that was designed for one channel condition is often used for a different one. We study the sensitivity of the above source-channel coding systems to channel fluctuations. We first show that a small change in the channel statistics can lead to a collapse of the systems if the error protection strategy is not updated. Then we answer the following important questions: How does a rate-optimal protection scheme change when the channel conditions change? Can one determine the smallest variation of the channel statistics that requires a change of an optimal protection strategy?

2. TRANSMISSION OVER BSC

We consider a joint source-channel coding system for a BSC that uses an embedded source coder to generate the compressed bitstream and then protects successive blocks of this bitstream with a family of channel codes having error detection and error correction capability [6]. All channel codes generate codewords of a fixed length L bits. Given these channel codes, let $\mathcal{R} = \{r_1, \ldots, r_m\}$ be the set of corresponding code rates with $r_1 < \cdots < r_m$. We suppose that $p(r_1, \epsilon) < \cdots < p(r_m, \epsilon) < 1$, where $p(r_j, \epsilon),\ j = 1, \ldots, m$, is the probability of a decoding error in a packet of length L for code rate r_j and channel bit error rate (BER) ϵ. At the receiver, if an error is detected, the decoding is stopped, and the image is reconstructed from the correctly decoded packets received until that point.

Let N be the number of packets sent. An *N-packet error protection scheme* (EPS) $R = (r_{k_1}, \ldots, r_{k_N}) \in \mathcal{R}^N$ assigns to each packet $i,\ i = 1, \ldots, N$, a channel rate r_{k_i}. For $i = 1, \ldots, N-1$, $P_i(R, \epsilon) = p(r_{k_{i+1}}, \epsilon) \prod_{j=1}^{i} (1 - p(r_{k_j}, \epsilon))$ is the probability that no errors occur in the first i packets, with an error in the next one, and $P_N(R, \epsilon) = \prod_{j=1}^{N} (1 - p(r_{k_j}, \epsilon))$ is the probability that no errors occur in the N packets. An N-packet EPS is *ϵ-optimal* if it maximizes the expected number of correctly decoded source bits

$$E_N^{(\epsilon)}(r_{k_1}, \ldots, r_{k_N}) = \sum_{i=1}^{N} P_i(R, \epsilon) \sum_{j=1}^{i} v(r_{k_j}), \qquad (1)$$

where $v(r_{k_j})$ is the number of source bits in packet j. If $R = (r_1^*, \ldots, r_N^*)$ is an ϵ-optimal EPS, then $r_1^* \leq \cdots \leq r_N^*$ [7].

In the following, we make the reasonable assumption that if ϵ_1 and ϵ_2 are two BERs with $\epsilon_1 > \epsilon_2$, then for $r_i > r_j$

$$p(r_i, \epsilon_1) - p(r_i, \epsilon_2) \geq p(r_j, \epsilon_1) - p(r_j, \epsilon_2). \qquad (2)$$

The following proposition explains how to update an optimal EPS when the BER of the BSC changes.

Proposition 1 *Let N be the number of packets sent and let ϵ be a BER of the BSC. Suppose that $(r_1^*, \ldots, r_N^*)$ is an ϵ-optimal EPS. Suppose now that the BER of the BSC is $\epsilon' > \epsilon$. Then if $(s_1, \ldots, s_N)$ is an ϵ'-optimal EPS, we must have $s_i \leq r_i^*$ for all $i = 1, \ldots, N$.*

The proof of the proposition is given in the appendix. The result of the proposition was expected since one should generally use a stronger protection when the BER increases. The proposition can be extended in a straightforward way to the case where the BER decreases.

An important consequence of the proposition is that we expect an increase of the BER to be more harmful than a decrease of the BER. Indeed, in an embedded bitstream, the bits have decreasing importance. Thus, compared to a BER decrease, a BER increase will generally leave more sensitive packets with a wrong protection. Therefore, in situations where on-line estimations of the channel statistics are not possible, it is recommended to design the system for the highest possible BER.

Because the number of EPSs is finite, Proposition 1 implies that for each BER ϵ, there exists a positive number $\Delta\epsilon$ such that the optimal EPS is constant in the interval $[\epsilon, \epsilon+\Delta\epsilon)$. We now provide an algorithm to compute $\Delta\epsilon$. We need first some notation.

Since $r_1^* \leq \cdots \leq r_N^*$, one can write the ϵ-optimal EPS $(r_1^*, \ldots, r_N^*)$ as $(\underbrace{r_{q_1}, \ldots, r_{q_1}}_{t_1}, \underbrace{r_{q_2}, \ldots, r_{q_2}}_{t_2}, \ldots, \underbrace{r_{q_K}, \ldots, r_{q_K}}_{t_K})$, where $r_{q_i} \neq r_{q_j}$ for $i \neq j$, $t_1, \ldots, t_K$ are positive integers, and $K \leq N$. With this notation, let

$$\begin{aligned} L_n(r_l, \delta) \quad = \quad & v(r_{N-i}^*)(p(r_{N-i}^*, \epsilon+\delta) - p(r_{N-i}^*, \epsilon)) - \\ & v(r_l)(p(r_l, \epsilon+\delta) - p(r_l, \epsilon)) + \\ & E_i^{(\epsilon+\delta)}(r_{N-i+1}^*, \ldots, r_N^*)(p(r_{N-i}^*, \epsilon+\delta) - \\ & p(r_l, \epsilon+\delta)) + E_1^{(\epsilon)}(r_l) - E_1^{(\epsilon)}(r_{N-i}^*), \end{aligned}$$

where $n = 1, \ldots, K$, $i = \sum_{j=n}^{K} t_j - 1$, and $l = 1, \ldots, m$. Then $\Delta\epsilon$, the smallest increase of BER that causes a change of the ϵ-optimal EPS, can be computed as follows.

Algorithm 1 *Using the above notations:*
1. Set $n = K$ and $r_{q_0} = r_1$.
2. Set $\Delta\epsilon_n = \infty$. If $n = 0$, set $\Delta\epsilon = \min_{j=1,\ldots,K} \Delta\epsilon_j$ and stop.
3. Set $r = r_{q_n}$.
4. If $r > r_{q_{n-1}}$, set r to the first smaller code rate in $\mathcal{R}$; otherwise go to Step 8.
5. Let δ be the smallest positive real number that gives $L_n(r, \delta) > 0$; if such a number does not exist, set $\delta = \infty$.
6. If $\Delta\epsilon_n > \delta$, then set $\Delta\epsilon_n = \delta$.
7. Go to Step 4.
8. Set $n = n - 1$ and go to Step 2.

The proof of correctness of the algorithm follows from Proposition 1, the results of [7], and the observation that $L_n(r_l, \delta) > 0$ is equivalent to

$$E_{i+1}^{(\epsilon+\delta)}(r_l, r_{N-i+1}^*, \ldots, r_N^*) > E_{i+1}^{(\epsilon+\delta)}(r_{N-i}^*, \ldots, r_N^*). \quad (3)$$

Note that the algorithm returns $\Delta\epsilon = \infty$ if the optimal scheme does not change when the BER increases. The algorithm is very fast because in the worst case Km steps are needed and $K \leq \min(N, m)$.

The algorithm has a great practical importance because once $\Delta\epsilon$ is known, one need not update the optimal EPS for all BER increases that are smaller than $\Delta\epsilon$. The algorithm can easily be adjusted to the case where the BER decreases.

3. PACKET ERASURE CHANNELS

In this section, we consider the joint source-channel coding system of [4, 3, 9]. The source coder produces a scalable bitstream, which is protected and sent as N packets of L symbols each. The channel coder builds L segments $S_1, \ldots, S_L$, each of which consists of $m_i \in \{1, \ldots, N\}$ information symbols, and protects each segment S_i with an (N, m_i) systematic RS code. If at most n packets of N are lost, then all segments that contain at most $N - n$ source symbols can be recovered. Thus, by adding the constraint $m_1 \leq m_2 \leq \cdots \leq m_L$, if at most $N - m_i$ packets are lost, then the decoder can recover at least the first i segments. Let $\mathcal{F}$ be a set of L-tuples $(f_1, \ldots, f_L)$, where $f_i = N - m_i$, $1 \leq i \leq L$, and $f_1 \geq f_2 \cdots \geq f_L$. Then, for an RS protection $F = (f_1, \ldots, f_L) \in \mathcal{F}$, the expected number of received source symbols at packet erasure rate e is

$$E_L^{(e)}(f_1, \ldots, f_L) = \sum_{i=1}^{L} (N - f_i) P(f_i, e), \quad (4)$$

where $P(f_i, e)$ is the probability that at least f_i packets are received if the packet erasure rate is e. We call an RS protection that maximizes (4), an *e-optimal RS protection*. In [9], we show that an e-optimal RS protection is given by the equal error protection $(f_r, \ldots, f_r)$, where

$$f_r = \arg \max_{i=0,\ldots,N-1} (N - i) P(i, e). \quad (5)$$

Let e_1 and e_2 be two packet erasure rates such that $e_1 > e_2$. Similarly to (2), we assume that for $0 \leq i < j < N$

$$P(i, e_2) - P(i, e_1) \geq P(j, e_2) - P(j, e_1). \quad (6)$$

The following proposition, whose proof is given in the appendix, explains how an optimal protection changes when the channel conditions change.

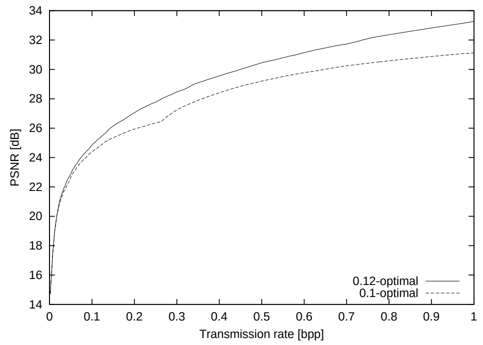
Proposition 2 *Let N be the number of sent packets and let e be an average packet erasure rate. Suppose that $(f_r, \ldots, f_r)$ is an e-optimal RS protection. Then for all $e' > e,\ e' < 1$, an e'-optimal RS protection is given by $(f_q, \ldots, f_q)$, where $f_r \leq f_q < N$.*

The result of this proposition was also expected. Indeed, when the packet erasure rate increases, the optimal protection should be stronger. By analogy with Algorithm 1, we can easily derive from Proposition 2 a method for computing the smallest increase of the packet erasure rate for which the optimal RS protection changes. Due to the lack of space we do not give the pseudocode of the algorithm, but we present its results in the next section.

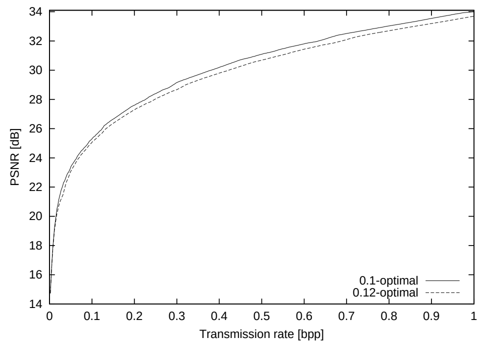
4. EXPERIMENTAL RESULTS

In all experiments, we used the SPIHT coder [2] and the 8 bits per pixel (bpp) grey-scale 512 × 512 Lenna image. We obtained similar results with the Goldhill image.

First we present results for the BSC case. The source code was protected with a concatenation of a 16-bit CRC code and an RCPC code. The RCPC coder [10] had mother code memory length six, generator polynomial (147,163,135,135) octal, and code rate $1/4$. The puncturing period was eight. The set of RCPC code rates was $\{8/9, 8/10, 8/12, \ldots, 8/24, 8/25, \ldots, 8/32\}$. The packet length



(a)



(b)

Fig. 1. Expected PSNR vs. transmission rate for the 0.12-optimal EPS and the 0.1-optimal EPS at (a) BER $= 0.12$ (b), BER $= 0.1$.

ϵ	$\Delta\epsilon$
0.01	0.001611
0.011611	0.000002
0.011613	0.000003
0.011616	0.000002
0.011618	0.000002

Table 1. Smallest BER increase $\Delta\epsilon$ that changes the optimal EPS.

was $L = 512$ bits. The decoder was based on a list Viterbi algorithm. Figure 1 compares the expected peak-signal-to-noise ratio (PSNR) of a 0.1 and 0.12-optimal EPSs for various transmission rates. Note that at BER $= 0.12$ and transmission rate 1 bpp, the 0.12-optimal EPS outperformed the 0.1-optimal EPS by more than 2 dBs. Thus, using the 0.1-optimal EPS at BER $= 0.12$ caused a significant performance loss. On the other hand, at BER $= 0.1$, the 0.12-optimal EPS showed a small performance loss compared to the 0.1-optimal EPS. This is in accordance with the remark after Proposition 1 that worse results are expected when the BER increases.

Figure 2 shows the performance of a 0.05-optimal EPS at transmission rate 1 bpp when the BER was varied from 0.01 to 0.12. For BERs higher than 0.05, the expected received source rate rapidly decreased, reaching zero for the highest BERs.

Table 1 shows for some BERs ϵ, the smallest increase that caused a change of the optimal EPS. For each $r_i \in \mathcal{R}$, we modeled $p(r_i, \epsilon)$ as a third order polynomial of ϵ and used Algorithm 1 to compute $\Delta\epsilon$. We obtained similar results for other BERs.

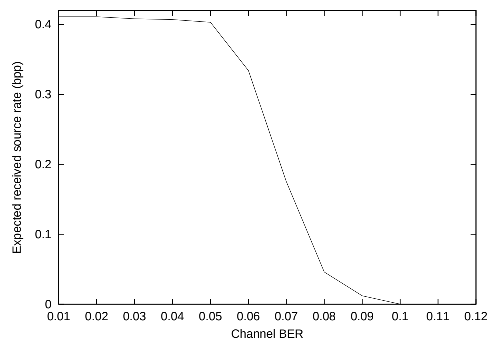


Fig. 2. Expected received source rate (in bpp) for a 0.05-optimal EPS at transmission rate 1 bpp as a function of the BER.

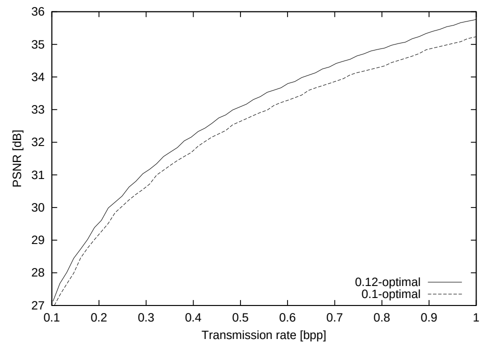


Fig. 3. Expected PSNR vs. transmission rate for the 0.12-optimal and 0.1-optimal RS protections. The packet erasure rate is 0.12.

The next experiments are for packet erasure channels. Figure 3 compares the performance of a 0.12 and 0.1-optimal RS protection when the packet erasure rate was 0.12. Figure 4 shows the performance of a 0.05-optimal RS protection as a function of the packet erasure rate. The results show that a channel mismatch can have a fatal effect on the performance of the system.

Figure 5 shows the smallest increase of the erasure rate that changed the optimal RS protection at transmission rate 0.146 bpp. The packet loss probability function was modeled as an exponentially decreasing function of the number of lost packets [4, 9].

We also studied the sensitivity of a distortion-optimal protection for packet erasure channels. In contrast to the BSC case, the performance of a distortion-optimal solution can be much better than that of a rate-optimal solution [9]. When the erasure rate varied, the distortion-optimal solution changed more frequently than a rate-optimal one. However, by updating a distortion-optimal protection only when the rate-optimal one changed, the quality loss did not exceed 0.01 dB in expected PSNR. Thus, the behavior of a rate-optimal solution was a good indication for that of a distortion-optimal protection.

5. CONCLUSION

We studied the influence of channel changes on optimal error protection of two popular joint source-channel coding systems. Note

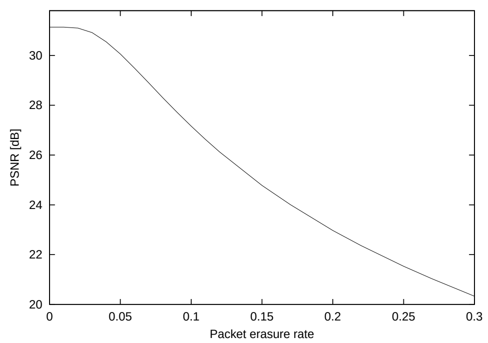


Fig. 4. Expected PSNR of a 0.05-optimal RS protection as a function of the erasure rate.

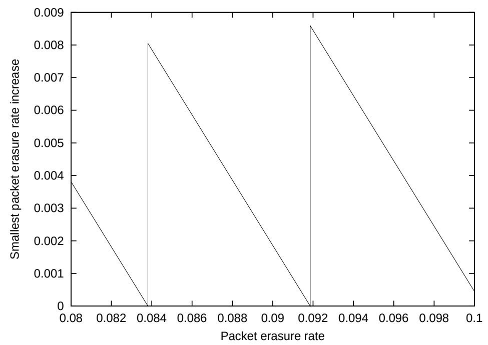


Fig. 5. Smallest increase of the packet erasure rate that caused the change of the optimal RS protection.

that the proposed techniques are also applicable to scalable video transmission. Future work may be the extension of the results to distortion-based optimal protection [6, 9].

6. APPENDIX

Proof of Proposition 1. Suppose that there exists $i \in \{0, \ldots, N-1\}$ such that $s_{N-i} > r^*_{N-i}$. Since $(s_1, \ldots, s_N)$ is ϵ'-optimal, we know that $(s_{N-i}, \ldots, s_N)$ is also ϵ'-optimal [8]. Thus,

$$E^{(\epsilon')}_{i+1}(s_{N-i}, \ldots, s_N) \geq E^{(\epsilon')}_{i+1}(r^*_{N-i}, \ldots, r^*_N), \qquad (7)$$

which, can be rewritten (see [7, 8])

$$E^{(\epsilon')}_1(r^*_{N-i}) - E^{(\epsilon')}_1(s_{N-i}) \leq E^{(\epsilon')}_i(r^*_{N-i+1}, \ldots, r^*_N)(p(r^*_{N-i}, \epsilon') - p(s_{N-i}, \epsilon')). \quad (8)$$

Inequality (8) can be expressed as

$$E^{(\epsilon)}_1(r^*_{N-i}) - E^{(\epsilon)}_1(s_{N-i}) \leq v(r^*_{N-i})(p(r^*_{N-i}, \epsilon') - p(r^*_{N-i}, \epsilon)) - v(s_{N-i})(p(s_{N-i}, \epsilon') - p(s_{N-i}, \epsilon)) + E^{(\epsilon')}_i(r^*_{N-i+1}, \ldots, r^*_N)(p(r^*_{N-i}, \epsilon') - p(s_{N-i}, \epsilon')). \quad (9)$$

Suppose first that $i \leq t_K$ (see the notation after Proposition 1). Then the left-hand side of (9) is nonnegative. But $s_{N-i} > r^*_{N-i}$ gives

$$p(r^*_{N-i}, \epsilon') - p(s_{N-i}, \epsilon') < 0$$

and because of (2)

$$v(s_{N-i})(p(s_{N-i}, \epsilon') - p(s_{N-i}, \epsilon)) > v(r^*_{N-i})(p(r^*_{N-i}, \epsilon') - p(r^*_{N-i}, \epsilon)).$$

The right-hand side is therefore always negative, and thus, (9) cannot be fulfilled. Using the same approach we can prove that $s_{N-i} \leq r^*_{N-i}$ when $i > t_K$.

Proof of Proposition 2. Let $\delta P(f_i) = P(f_i, e) - P(f_i, e')$, where $f_i \in \{f_r, f_q\}$. Since $(f_r, \ldots, f_r)$ is e-optimal, we have from (5)

$$T_1 = (N - f_r)P(f_r, e) - (N - f_q)P(f_q, e) \geq 0.$$

Similarly, because $(f_q, \ldots, f_q)$ is e'-optimal, we have

$$T_2 = (N - f_r)P(f_r, e') - (N - f_q)P(f_q, e') \leq 0. \qquad (10)$$

But

$$T_2 = T_1 - (N - f_r)\delta P(f_r) + (N - f_q)\delta P(f_q).$$

Thus, if $f_q < f_r$, (6) gives $T_2 > 0$, which contradicts (10).

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