

Fast Algorithm for Rate-Based Optimal Error Protection of Embedded Codes

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Abstract

Embedded image codes are very sensitive to channel noise because a single bit error can lead to an irreversible loss of synchronization between the encoder and the decoder. Sherwood and Zeger introduced a powerful system that protects an embedded wavelet image code with a concatenation of a cyclic redundancy check coder for error detection and a rate-compatible punctured convolutional coder for error correction. For such systems, Chande and Farvardin proposed an unequal error protection strategy that maximizes the expected number of correctly received source bits subject to a target transmission rate. Noting that an optimal strategy protects successive source blocks with the same channel code, we give an algorithm that accelerates the computation of the optimal strategy of Chande and Farvardin by finding an explicit formula for the number of occurrences of a same channel code. Experimental results with two competitive channel coders and a binary symmetric channel showed that the speed-up factor over the approach of Chande and Farvardin ranged from 2.82 to 44.76 for transmission rates between 0.25 and 2 bits per pixel.

Index Terms

Image coding, joint source-channel coding, forward error correction, image communication.

I. INTRODUCTION

We consider a joint source-channel coding system for progressive image transmission that uses an embedded source coder and a finite set of channel codes with error detection and error protection capability [1], [2]. For example, the source coder may be the SPIHT coder [3], JPEG2000 [4], or 3D SPIHT [5], and the channel coder may consist of the concatenation of an outer cyclic redundancy check (CRC) coder for error detection and an inner rate-compatible punctured convolutional (RCPC) coder for error correction [1]. The channel encoder transforms successive blocks of the source bitstream into a sequence of channel codewords, which are sent over a memoryless noisy channel. When the first decoding error is detected, the decoding is stopped, and the image is reconstructed from the correctly decoded codewords received up to that point. This is a reasonable approach for many embedded codes, including the SPIHT and JPEG2000 source codes, where, generally, a single bit error leads to a loss of synchronization between the encoder and the decoder [6]. In [1], [2], the size of the blocks of information bits is fixed, while the channel codewords have a variable size. But, for many applications, including transmission in ATM networks, it is more suitable to fix the size of the channel codewords and let the information blocks have a variable size (see Fig. 1).

A challenging problem is to determine an optimal error protection for this system; that is, an assignment of the available channel codes (each specified by a different code rate) to the successive source blocks that minimizes the expected distortion subject to a target transmission rate. An alternative is to maximize the expected number of correctly received source bits for this target transmission rate. Though the solution based on maximizing the expected source rate is suboptimal in the distortion sense, the loss in reconstruction quality is small for efficient embedded coders (for a comparison, see [7], [2] for the original system of [1] and [8] for the system considered in this paper). Moreover, the rate-based optimization has three advantages. First, an optimal protection can be found in linear-time by adapting the algorithm proposed by Chande and Farvardin in the fixed-length information block framework (Algorithm 2 in [2]) to the fixed-length channel codeword one. Second, this optimal protection is independent of both the source coder and the test image. Thus, it can also be implemented at the receiver side, avoiding the need for overhead information. Third, if an optimal protection for a target transmission rate is known, then an optimal solution for all lower transmission rates can easily be derived without repeating the optimization.

The strategy of Chande and Farvardin determines successively the optimal code rate for each source block, starting from the last one. We propose an algorithm that reduces the time complexity of this strategy. The basic idea of our method is to see that an optimal solution shows long runs of the same code rate, which is typically due to the fact that the number of available code rates is much lower than the number of transmitted codewords and to the property that in an optimal solution the code rates are nondecreasing. Thus one can save computation time by finding an explicit formula for the length of the code rate runs. For example, in Fig. 1 the solution has three runs of respective length two, four, and three.

The paper is organized as follows. In Section II, we introduce our terminology, adapt the algorithm of Chande and Farvardin (Algorithm 2 in [2]) to the fixed-length channel codeword context, and provide some properties of a rate-based optimal protection. In Section III, we give our algorithm and prove its optimality. In Section IV, we provide experimental results that illustrate the speed-up obtained with our approach for a binary symmetric channel and two different channel coders.

II. RATE-BASED OPTIMAL ERROR PROTECTION

We consider a joint source-channel coding system where the source coder generates an embedded bitstream and the channel coder is given by a family $\mathcal{C} = \{C_1, \dots, C_m\}$ of channel codes with error detection and error correction capability. These codes are such that all channel codewords have the same size L (in bits). The set of corresponding code rates is denoted by $\mathcal{R} = \{r_1, \dots, r_m\}$, where

$r_1 < \dots < r_m$. We assume without loss of generality that $p(r_1) < \dots < p(r_m) < 1$, where $p(r_j)$, $j = 1, \dots, m$, is the probability of a decoding error when channel code C_j is used. The system transforms successive blocks b_1, b_2, \dots of the embedded source bitstream into a sequence of channel codewords $c(b_1), c(b_2), \dots$. The number of information bits in a block protected by code rate r_j is thus $v(r_j) = \lfloor Lr_j \rfloor$. Each channel packet consists of a single channel codeword (for simplicity, we ignore the header of the channel packet). For a given transmission bit budget B , the number of packets sent is $N = \lfloor \frac{B}{L} \rfloor$. An N -packet error protection scheme (EPS) $R = (r_{k_1}, \dots, r_{k_N}) \in \mathcal{R}^N$ protects the i th source block b_i , $i = 1, \dots, N$, with a channel code rate $r_{k_i} \in \mathcal{R}$. For example, Fig. 1 shows a nine-packet EPS $(1/4, 1/4, 1/2, 1/2, 1/2, 1/2, 2/3, 2/3, 2/3)$.

The packets $c(b_1), c(b_2), \dots, c(b_N)$ are sent successively over a memoryless noisy channel. Thus, we do not consider multiplexing and interleaving. As soon as a decoding error is detected, the decoding is stopped, and the image is reconstructed from the packets correctly decoded up to that point. Here we suppose that the probability of an undetected error is equal to zero. For $i = 1, \dots, N - 1$, the number

$$P_i(R) = p(r_{k_{i+1}}) \prod_{j=1}^i (1 - p(r_{k_j}))$$

is the probability that no decoding errors occur in the first i packets, with an error in the next one, $P_0(R) = p(r_{k_1})$ is the probability that a decoding error occurs in the first packet, and $P_N(R) = \prod_{j=1}^N (1 - p(r_{k_j}))$ is the probability that no decoding errors occur in the N packets. For an N -packet EPS $R = (r_{k_1}, \dots, r_{k_N}) \in \mathcal{R}^N$ the expected distortion is

$$E_N[d](R) = \sum_{i=0}^N P_i(R) d_i(R), \quad (1)$$

where d_0 is a constant, and for $i \geq 1$, $d_i(R)$ is the distortion from the reconstruction using the first i information blocks.

Because minimizing $E_N[d]$ over \mathcal{R}^N is time consuming, suboptimal solutions have been proposed [9], [8]. For example, one can replace the original problem by that of maximizing the expected number of correctly decoded source bits, which is

$$E_N(R) = \sum_{i=1}^N P_i(R) V_i(R), \quad (2)$$

where $V_i(R) = \sum_{j=1}^i v(r_{k_j})$ is the number of information bits in the first i packets. For simplicity of notation, we will sometimes drop the subscript N from E_N when the number of packets is clear from the context. An N -packet EPS $T \in \mathcal{R}^N$ is *rate-optimal* if for any $R \in \mathcal{R}^N$ we have $E_N(T) \geq E_N(R)$.

Chande and Farvardin [2] found a linear-time algorithm that computes a rate-optimal EPS for the original system of [1]. We now present this algorithm for the system considered in this paper (Fig. 1 (a)). Before presenting the algorithm, we provide several important results.

Lemma 1: Let $(r_{k_1}, r_{k_2}, \dots, r_{k_N}) \in \mathcal{R}^N$ be an N -packet EPS. Then

$$\begin{aligned} E_N(r_{k_1}, r_{k_2}, \dots, r_{k_N}) &= (1 - p(r_{k_1}))(v(r_{k_1}) + E_{N-1}(r_{k_2}, \dots, r_{k_N})) \\ &= E_1(r_{k_1}) + (1 - p(r_{k_1}))E_{N-1}(r_{k_2}, \dots, r_{k_N}). \end{aligned} \quad (3)$$

Proof. See Appendix I.

The above lemma gives a recurrence relation between $E_N(r_{k_1}, r_{k_2}, \dots, r_{k_N})$ and $E_{N-1}(r_{k_2}, \dots, r_{k_N})$.

Lemma 2: If the $(N - 1)$ -packet EPS (r_2^*, \dots, r_N^*) is rate-optimal and if

$$E_N(r_1^*, r_2^*, \dots, r_N^*) \geq E_N(r_k, r_2^*, \dots, r_N^*)$$

for all $r_k \in \mathcal{R}$, then the N -packet EPS (r_1^*, \dots, r_N^*) is rate-optimal.

Proof. See Appendix II.

This lemma shows how to derive a rate-optimal N -packet EPS from a rate-optimal $(N - 1)$ -packet EPS.

Proposition 1: If the N -packet EPS (r_1^*, \dots, r_N^*) is rate-optimal, then the following holds.

- (a) For $1 \leq i \leq N - 1$ the $(N - i)$ -packet EPS $(r_{i+1}^*, \dots, r_N^*)$ is rate-optimal.
- (b) $E_i(r_{N-i+1}^*, \dots, r_{N-1}^*, r_N^*)$ is an increasing function of i , $i = 1, \dots, N$.
- (c) $r_1^* \leq \dots \leq r_N^*$.
- (d) Let $1 < k \leq N$. If the $(N - k + 1)$ -packet EPS $(r_{j_k}, r_{j_{k+1}}, \dots, r_{j_N})$ is rate-optimal, then $r_{k-1}^* \leq r_{j_k}$.

Proof. See Appendix III.

The most important result is part (c) of the proposition which states that in a rate-optimal EPS the code rates are nondecreasing with the packet number. Part (a) states that once a rate-optimal N -packet EPS is known, a rate-optimal k -packet ($k < N$) EPS can easily be derived by truncating the first $(N - k)$ code rates. Part (d), which extends part (c), is useful when a rate-optimal solution is not unique. The following example illustrates the importance of this result. Suppose that $\mathcal{R} = \{r_1, r_2, r_3, r_4\}$. Suppose that both (r_2, r_4, r_4) and (r_3, r_4, r_4) are rate-optimal. Let (r_1^*, \dots, r_4^*) be any rate-optimal four-packet EPS. Then $r_1^* \leq \min(r_2, r_3) = r_2$.

Note that (b) is not necessarily satisfied if the EPS is not rate-optimal.

Lemma 1, Lemma 2, and Proposition 1(c) can be found in [2]. However, no proofs were provided, and all results were given for the particular system considered there.

To find a rate-optimal N -packet EPS the approach of Chande and Farvardin consists of determining first a rate-optimal one-packet EPS and to complete the solution by repetitive use ($N - 1$ times) of Lemma 2 together with Proposition 1(c). This gives the following algorithm.

Proposition 2: Let N be a positive integer and let \mathcal{R} be the set of m code rates $r_1 < \dots < r_m$. A rate-optimal N -packet EPS (r_1^*, \dots, r_N^*) can be computed as follows.

1. Set $i = 1$ and $j_1 = \arg \max_{k=1, \dots, m} E_1(r_k)$.
2. If $i = N$, then set $(r_1^*, \dots, r_N^*) = (r_{j_1}, \dots, r_{j_1})$ and stop. If $r_{j_i} = r_1$, then set $(r_1^*, \dots, r_N^*) = (r_{j_i}, \dots, r_{j_i}, r_{j_{i-1}}, \dots, r_{j_i})$ and stop.
3. Set $i = i + 1$, $j_i = \arg \max_{k=1, \dots, m, r_k \leq r_{j_{i-1}}} E_i(r_k, r_{j_{i-1}}, \dots, r_{j_i})$, and go to Step 2.

III. FAST ALGORITHM FOR RATE-OPTIMAL PROTECTION

In this section, we show how to accelerate the algorithm of Proposition 2. We first introduce some notations and technical lemmas that are needed to prove the optimality of our algorithm.

For $r_j, r_k \in \mathcal{R}$ and integer $t \geq 1$, let $q(r_j) = 1 - p(r_j)$,

$$T(r_j, t) = \frac{E_1(r_j)(1 - q(r_j)^t)}{1 - q(r_j)},$$

and

$$M(r_k, r_j) = \frac{E_1(r_k)(1 - q(r_j)) - E_1(r_j)(1 - q(r_k))}{(q(r_k) - q(r_j))(1 - q(r_j))}.$$

Then we have:

Lemma 3: Let $N \geq 1$. Then for any positive integers t_0, \dots, t_i , $N = t_0 + \dots + t_i$, and channel code rates $r_{j_0}, \dots, r_{j_i} \in \mathcal{R}$, $p(r_{j_k}) \neq 0$, $0 \leq k \leq i$

$$\begin{aligned} E_N(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t_i}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) &= \sum_{l=0}^i E_{t_l}(r_{j_l}, \dots, r_{j_l}) \prod_{k=l+1}^i q(r_{j_k})^{t_k} \\ &= \sum_{l=0}^i T(r_{j_l}, t_l) \prod_{k=l+1}^i q(r_{j_k})^{t_k}. \end{aligned}$$

Proof. See Appendix IV.

Let $r_{j_0}, \dots, r_{j_i} \in \mathcal{R}$ with $p(r_{j_i}) \neq 0$ and let t_0, \dots, t_{i-1} be positive integers. Let $A_i = \frac{E_1(r_{j_i})}{1 - q(r_{j_i})}$, $B_0 = 0$, and

$$B_i = E_{t_0 + \dots + t_{i-1}}(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$$

for $i \geq 1$. For $k \in \{1, \dots, m\}$, $t \in \mathbb{R}$, and $i \geq 0$, let

$$\psi_{i,k}(t) = (A_i - B_i)q(r_{j_i})^{t-1} - M(r_k, r_{j_i}).$$

Lemma 4: For $i \geq 0$, let $r_{j_0}, \dots, r_{j_i} \in \mathcal{R}$ with $p(r_{j_i}) \neq 0$ and let t_0, \dots, t_{i-1} be positive integers. Suppose that $k \in \{1, \dots, m\}$ and $r_k < r_{j_i}$. Then $\psi_{i,k}(t)$ and

$$E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) - E(\underbrace{r_k, r_{j_i}, \dots, r_{j_i}}_{t-1}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$$

have the same sign for any positive integer t .

Proof. See Appendix V.

Lemma 5: For $i \geq 1$, let $r_{j_0}, \dots, r_{j_i} \in \mathcal{R}$ with $p(r_{j_i}) \neq 0$ and let t_0, \dots, t_{i-1} be positive integers. If $(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ is a rate-optimal EPS and if $\psi_{i-1,j_i}(t_{i-1} + 1) < 0$, then $A_i - B_i > 0$.

Proof. See Appendix VI.

We now introduce our algorithm. The basic idea is to compute the run length of a code rate ahead of time.

Proposition 3: Let N be a positive integer. Then a rate-optimal N -packet EPS is

$(\underbrace{r_{j_n}, \dots, r_{j_n}}_{t_n}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$, where $n \geq 0$, $r_{j_0}, \dots, r_{j_n} \in \mathcal{R}$, and $t_0, \dots, t_n \geq 1$ are given by the following algorithm.

1. Set $j_0 = \arg \max_{k=1, \dots, m} E_1(r_k)$ (if the argument is not unique, choose the one which corresponds to the smallest code rate). Set $i = 0$.
2. If $r_{j_i} = r_1$, set $n = i$, $t_n = N - \sum_{p=0}^{i-1} t_p$ and stop.
3. For all $k \in \{1, \dots, m\}$, $r_k < r_{j_i}$, if $a_{i,k} = \frac{\log \frac{M(r_k, r_{j_i})}{A_i - B_i}}{\log q(r_{j_i})} + 1$ exists and is finite, then set $t_{i,k} = \lfloor a_{i,k} \rfloor$. Otherwise, set $t_{i,k} = N - \sum_{p=0}^{i-1} t_p$. Let $t_i = \min_{k, r_k < r_{j_i}} t_{i,k}$.
4. If $N \leq \sum_{p=0}^i t_p$, set $n = i$, $r_{j_n} = r_{j_i}$, $t_n = N - \sum_{p=0}^{i-1} t_p$ and stop. Otherwise, set

$$j_{i+1} = \arg \max_{k, r_k < r_{j_i}, t_{i,k} = t_i} E(r_k, \underbrace{r_{j_i}, \dots, r_{j_i}}_{t_i}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$$

(if the argument is not unique, choose the one which corresponds to the smallest code rate), set $i = i + 1$ and go to Step 2.

Proof. If $r_{j_0} = r_1$, then the algorithm yields the EPS $(\underbrace{r_{j_0}, \dots, r_{j_0}}_N)$, which is rate-optimal by Lemma 2 and Proposition 1(c). Suppose now that $r_{j_0} \neq r_1$. Let $k \in \{1, \dots, m\}$ with $r_k < r_{j_0}$. Because $A_0 - B_0 = A_0 > 0$, the function $\psi_{0,k}$ is decreasing on \mathbb{R} from $+\infty$ to $-M(r_k, r_{j_0})$. If $a_{0,k} \notin \mathbb{R}$, then $t_{0,k} = N - \sum_{p=0}^{i-1} t_p = N \geq 1$. If $a_{0,k} \in \mathbb{R}$, then $\psi_{0,k}(1) \geq 0$ by Lemma 4. Moreover, $\psi_{0,k}(a_{0,k}) = 0$. Thus, $a_{0,k} \geq 1$, which gives $t_{0,k} \geq 1$. Hence, $t_0 \geq 1$. Now let t be an integer such that $1 \leq t \leq t_0$. If $a_{0,k} \in \mathbb{R}$, then $\psi_{0,k}(t) \geq \psi_{0,k}(t_0) \geq \psi_{0,k}(t_{0,k}) \geq \psi_{0,k}(a_{0,k}) = 0$. If $a_{0,k} \notin \mathbb{R}$, then $-M(r_k, r_{j_0}) \geq 0$ and

here also $\psi_{0,k}(t) \geq 0$. Thus, Lemma 4 gives

$$E(r_{j_0}, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t-1}) \geq E(r_k, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t-1}) \quad (4)$$

for all $k \in \{1, \dots, m\}$ with $r_k < r_{j_0}$. If $N \leq \sum_{p=0}^i t_p = t_0$, then the algorithm yields the EPS $(\underbrace{r_{j_0}, \dots, r_{j_0}}_N)$, which is rate-optimal by inequality (4), Lemma 2, and Proposition 1(c). We assume now that $N > \sum_{p=0}^i t_p = t_0$. We have $t_{0,j_1} = t_0$. Since $t_{0,j_1} + 1 > a_{0,j_1}$, we get $\psi_{0,j_1}(t_0 + 1) < \psi_{0,j_1}(a_{0,j_1}) = 0$, which gives

$$E(r_{j_1}, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) > E(\underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0+1}) \quad (5)$$

by Lemma 4 and $A_1 - B_1 > 0$ by Lemma 5. On the other hand $t_0 + 1 \leq a_{0,k}$ for all k such that $t_{0,k} \neq t_0$. Thus, $\psi_{0,k}(t_0 + 1) \geq \psi_{0,k}(a_{0,k}) = 0$, or

$$E(\underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0+1}) \geq E(r_k, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \quad (6)$$

for all k such that $t_{0,k} \neq t_0$. From inequalities (5) and (6), we obtain

$$E(r_{j_1}, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \geq E(r_k, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \quad (7)$$

for all k such that $t_{0,k} \neq t_0$. Moreover, by construction, inequality (7) holds for all k such that $r_k < r_{j_0}$ and $t_{0,k} = t_0$. Thus, inequality (7) is true for all k such that $r_k < r_{j_0}$. Using the same approach as for (4), we can prove that for $i = 1, 2, \dots, n$, if $1 \leq t \leq t_i$, then for all $r_k \leq r_{j_i}$

$$E_{t_0 + \dots + t_{i-1} + t}(r_{j_i}, \underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \geq E_{t_0 + \dots + t_{i-1} + t}(r_k, \underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}),$$

and with the same approach as for (7), we can show that for $i = 2, \dots, n$

$$E(r_{j_i}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \geq E(r_k, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$$

for $r_k \leq r_{j_{i-1}}$. Thus, Lemma 2 and Proposition 1(c) show that the EPS yielded by the algorithm is optimal. When in Step 1 or 4 the argument is not unique, we choose the smallest code rate because this choice reduces the complexity of the algorithm by excluding more code rates from the next optimization steps. Note that Proposition 1(d) justifies why the choice of the smallest code rate does not violate the optimality of the algorithm. \square

In Step 3, B_i should be computed using Lemma 3. In Step 4, $E(r_k, \underbrace{r_{j_i}, \dots, r_{j_i}}_{t_i}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ can easily be computed from B_i by Lemma 1. Finally, if in Step 3 there exists a unique k such that $\lfloor a_{i,k} \rfloor = t_i$, then $j_{i+1} = k$.

In Fig. 1, for example, the run lengths of the code rates are $t_0 = 3, t_1 = 4$, and $t_2 = 2$ with $r_{j_0} = 2/3, r_{j_1} = 1/2$, and $r_{j_2} = 1/4$.

IV. RESULTS

In this section, we compare the time complexity of our algorithm to that of Proposition 2 for a binary symmetric channel and two channel coders. We recall that N and L denote the number of packets sent and the length of the channel codeword, respectively. Thus, the transmission rate is $R_T = NL/n^2$ bits per pixel (bpp) for $n \times n$ images.

In the first experiment, the channel coder was a concatenation of a CRC-32 coder and a rate-compatible punctured turbo (RCPT) coder [10]. The generator polynomial of the CRC code was (32,26,23,22,16,12,11,10,8,7,5,4,2,1,0). The turbo coder consisted of two identical recursive systematic convolutional coders [11] with memory length four and generator polynomials (31, 27) octal. The code rate of the mother code was $20/60 = 1/3$, and the puncturing rate was 20, yielding 41 possible channel code rates. The length of a channel codeword was equal to $L = 2048$ bits, consisting of a variable number of source bits, 32 CRC bits, 4 bits to set the turbo encoder into a state of all zeros, and protection bits. We used iterative maximum a posteriori decoding, which was stopped if no correct sequence was found after 20 iterations. When the source coder is the SPIHT coder, this system yields state-of-the-art rate-distortion performance (see Table 2 in [12]). For each bit error rate (BER) and each code rate, the probability of a packet decoding error was computed with 50,000 Monte Carlo simulations. Channel code rates for which this probability was equal to one were removed. The set of used code rates was $\mathcal{R} = \{20/40, 20/38, 20/36, 20/35\}$ for BER = 0.05 and $\mathcal{R} = \{20/56, 20/54, 20/52, 20/51, 20/50, 20/48, 20/47, 20/46\}$ for BER = 0.1. Using the notation of Proposition 3, the rate-optimal solution for BER = 0.05 is given by $r_{j_0} = 20/35, t_0 = 56, r_{j_1} = 20/36, t_1 = 225$, and $r_{j_2} = 20/40, t_2 = N - (t_0 + t_1)$. For example, suppose that we want to send a 512×512 image at a transmission rate of 1 bpp. Then $N = 128$ packets have to be sent. Whereas the approach of Chande and Farvardin determines the optimal code rate for each of the 128 corresponding source blocks, our strategy needs only two steps. The first one identifies the run of 56 source blocks (starting from the end) that should be protected by code rate 20/35, and the second step finds the run of 72 source blocks that should be protected by code rate 20/36.

For BER = 0.1, the rate-optimal solution is given by $r_{j_0} = 20/48, t_0 = 11, r_{j_1} = 20/50, t_1 = 30, r_{j_2} = 20/52, t_2 = 365$, and $r_{j_3} = 20/56, t_3 = N - (t_0 + t_1 + t_2)$.

In the second experiment, the channel coder was a concatenation of a CRC-16 coder and an RCPC

coder. The generator polynomial of the CRC code was as in [1]. The RCPC codes, taken from [13], had a mother code with memory length six, generator polynomial (147, 163, 135, 135) octal, and code rate 1/4. The puncturing period was 8, yielding 25 possible channel code rates. The channel codeword length was $L = 512$ bits, consisting of information bits, 16 CRC detection bits, 6 bits for setting the convolutional encoder into a state of all zeros, and protection bits. The decoding was based on a trellis search technique with a list Viterbi algorithm [14]. If the path selected by the Viterbi decoder was not declared correct by the CRC test, the decoder selected the next best path. This procedure was repeated until the CRC test was passed or 100 paths were considered. For each BER and each code rate, the probability of a packet decoding error was computed with 100,000 Monte Carlo simulations. The set of used rates was $\mathcal{R} = \{8/20, 8/18, 8/16, 8/14, 8/12, 8/10\}$ for BER = 0.05 and $\mathcal{R} = \{8/28, 8/27, 8/26, 8/25, 8/24, 8/22, 8/20, 8/18\}$ for BER = 0.1. The rate-optimal solution for BER = 0.05 is given by $r_{j_0} = 8/14, t_0 = 4, r_{j_1} = 8/16, t_1 = 86, r_{j_2} = 8/18, t_2 = 5863$, and $r_{j_3} = 8/20, t_3 = N - (t_0 + t_1 + t_2)$. For BER = 0.1, the rate-optimal solution is given by $r_{j_0} = 8/20, t_0 = 3, r_{j_1} = 8/22, t_1 = 19, r_{j_2} = 8/24, t_2 = 113, r_{j_3} = 8/25, t_3 = 333, r_{j_4} = 8/27, t_4 = 494$ and $r_{j_5} = 8/28, t_5 = N - (t_0 + t_1 + t_2 + t_3 + t_4)$.

Fig. 2 compares the time complexity of our algorithm to that of Proposition 2 for various target transmission rates. The transmission rate is given for 512×512 images. The CPU time was measured on a 195 MHz MIPS R10000 processor of an SGI Origin200. The algorithm of Proposition 2 is already fast. For example, for RCPC codes at transmission rate 2 bpp and channel BER = 0.1, it found an optimal solution in 1.02 ms. Except for very low transmission rates, our algorithm was always faster (i.e., the speed-up factor was greater than one). Generally, the speed-up factor increased with the transmission rate. However, due to the complex formula for $a_{i,k}$, which is needed to compute the length of a code rate run, a drop in the speed-up factor happened each time our algorithm needed to compute a new code rate run. This also explains why our algorithm was slower at very low transmission rates. Indeed, when only a few packets are present, the cost for computing a code rate run length outweighs the cost of computing the code rate for each channel codeword. Also when the smallest code rate was selected, both algorithms stopped and no further speed-up occurred (see the extremity of the graph corresponding to BER = 0.1 in Fig. 2 (b)). Finally, the speed-up factor was higher for BER = 0.05 than for BER = 0.1 because there were fewer code rate changes with the lower BER.

V. CONCLUSION

We considered joint source-channel coding for progressive image transmission in memoryless noisy channels. We showed how to speed-up a strategy of Chande and Farvardin that assigns channel codes to the source blocks such that the expected number of correctly received source bits is maximized subject to a target transmission rate. Whereas the original algorithm must determine the optimal channel code for each source block, our method determines ahead of time the number of successive source blocks that should be protected with the same channel code. Though the original algorithm is already fast, accelerating it is especially desirable in two applications. The first one is when the channel conditions change during transmission and, consequently, a new optimal protection has to be computed online. The second one is when it is used as an initial step in the local search algorithm of [8], which can find a near-optimal solution to the problem of minimizing the expected distortion (1).

APPENDIX I

PROOF OF LEMMA 1

Let $R = (r_{k_1}, \dots, r_{k_N})$. Then

$$\begin{aligned} E_N(R) &= v(r_{k_1}) \sum_{i=1}^N P_i(R) + \sum_{i=2}^N P_i(R) \sum_{j=2}^i v(r_{k_j}) \\ &= v(r_{k_1})(1 - p(r_{k_1})) + \sum_{i=2}^N P_i(R) \sum_{j=2}^i v(r_{k_j}), \end{aligned}$$

and the result follows from the equality

$$\sum_{i=2}^N P_i(R) \sum_{j=2}^i v(r_{k_j}) = (1 - p(r_{k_1})) E_{N-1}(r_{k_2}, \dots, r_{k_N}).$$

APPENDIX II

PROOF OF LEMMA 2

Suppose that the $(N - 1)$ -packet EPS (r_2^*, \dots, r_N^*) is rate-optimal. Let $r_{k_1}, r_{k_2}, \dots, r_{k_N} \in \mathcal{R}$. Then

$$\begin{aligned} E_N(r_1^*, r_2^*, \dots, r_N^*) &\geq E_N(r_{k_1}, r_2^*, \dots, r_N^*) \\ &= (1 - p(r_{k_1}))(v(r_{k_1}) + E_{N-1}(r_2^*, \dots, r_N^*)) \\ &\geq (1 - p(r_{k_1}))(v(r_{k_1}) + E_{N-1}(r_{k_2}, \dots, r_{k_N})) \\ &= E_N(r_{k_1}, r_{k_2}, \dots, r_{k_N}), \end{aligned}$$

which shows that the N -packet EPS (r_1^*, \dots, r_N^*) is rate-optimal.

APPENDIX III

PROOF OF PROPOSITION 1

(a) Let $r_{k_2}, \dots, r_{k_N} \in \mathcal{R}$. Then $E_N(r_1^*, r_2^*, \dots, r_N^*) \geq E_N(r_1^*, r_{k_2}, \dots, r_{k_N})$. By Lemma 1 this inequality can be expressed as

$$E_1(r_1^*) + (1 - p(r_1^*))E_{N-1}(r_2^*, \dots, r_N^*) \geq E_1(r_1^*) + (1 - p(r_1^*))E_{N-1}(r_{k_2}, \dots, r_{k_N})$$

or $E_{N-1}(r_2^*, \dots, r_N^*) \geq E_{N-1}(r_{k_2}, \dots, r_{k_N})$. Similarly, $E_{N-i}(r_{i+1}^*, \dots, r_N^*) \geq E_{N-i}(r_{k_{i+1}}, \dots, r_{k_N})$ for $i = 2, \dots, N-1$.

(b) By (a), the $(i+1)$ -packet EPS $(r_{N-i}^*, \dots, r_{N-1}^*, r_N^*)$ is rate-optimal. Thus, for $r_j \in \mathcal{R}$

$$\begin{aligned} E_{i+1}(r_{N-i}^*, \dots, r_{N-1}^*, r_N^*) &\geq E_{i+1}(r_{N-i+1}^*, \dots, r_{N-1}^*, r_N^*, r_j) \\ &= E_i(r_{N-i+1}^*, \dots, r_{N-1}^*, r_N^*) + E(r_j) \prod_{k=0}^{i-1} (1 - p(r_{N-k}^*)) \\ &> E_i(r_{N-i+1}^*, \dots, r_{N-1}^*, r_N^*), \end{aligned} \quad (8)$$

where (8) is obtained by successive use of (3).

(c) One can express (r_1^*, \dots, r_N^*) as $(\underbrace{r_{j_n}, \dots, r_{j_n}}_{t_n}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ with $n \geq 0, t_i \geq 1$ for $i = 0, \dots, n, N = \sum_{i=0}^n t_i$, and $r_{j_i} \neq r_{j_{i+1}}$, $0 \leq i \leq n-1$. Due to (a), we have $E_{t_0+1}(r_{j_1}, r_{j_0}, \dots, r_{j_0}) \geq E_{t_0+1}(r_{j_0}, r_{j_0}, \dots, r_{j_0})$, which can be written $E_1(r_{j_0}) - E_1(r_{j_1}) \leq (p(r_{j_0}) - p(r_{j_1}))E_{t_0}(r_{j_0}, \dots, r_{j_0})$. Thus, since $E_1(r_{j_0}) \geq E_1(r_{j_1})$, we get $p(r_{j_1}) < p(r_{j_0})$ and hence $r_{j_1} < r_{j_0}$. Similarly, the inequality $E_{t_0+1}(r_{j_1}, r_{j_0}, \dots, r_{j_0}) \geq E_{t_0+1}(r_{j_2}, r_{j_0}, \dots, r_{j_0})$ gives

$$E_1(r_{j_1}) - E_1(r_{j_2}) \geq (p(r_{j_1}) - p(r_{j_2}))E_{t_0}(r_{j_0}, \dots, r_{j_0}) \quad (9)$$

and $E_{t_0+t_1+1}(r_{j_1}, r_{j_1}, \dots, r_{j_1}, r_{j_0}, \dots, r_{j_0}) \leq E_{t_0+t_1+1}(r_{j_2}, r_{j_1}, \dots, r_{j_1}, r_{j_0}, \dots, r_{j_0})$ gives

$$E_1(r_{j_1}) - E_1(r_{j_2}) \leq (p(r_{j_1}) - p(r_{j_2}))E_{t_0+t_1}(r_{j_1}, \dots, r_{j_1}, r_{j_0}, \dots, r_{j_0}). \quad (10)$$

From (b), we have $E_{t_0+t_1}(r_{j_1}, \dots, r_{j_1}, r_{j_0}, \dots, r_{j_0}) > E_{t_0}(r_{j_0}, \dots, r_{j_0})$. Thus, inequalities (9) and (10) can be simultaneously fulfilled only when $r_{j_2} < r_{j_1}$. The same method can be used to show that $r_{j_{i+1}} < r_{j_i}$ for $2 \leq i \leq N-1$.

(d) We have

$$\begin{aligned} E(r_1^*, \dots, r_{k-1}^*, r_{j_k}, r_{j_{k+1}}, \dots, r_{j_N}) &= E(r_1^*, \dots, r_{k-1}^*) + E(r_{j_k}, \dots, r_{j_N}) \prod_{i=1}^{k-1} (1 - p(r_i^*)) \\ &\geq E(r_1^*, \dots, r_{k-1}^*) + E(r_k^*, \dots, r_N^*) \prod_{i=1}^{k-1} (1 - p(r_i^*)) \\ &= E(r_1^*, r_2^*, \dots, r_N^*). \end{aligned}$$

Thus the N -packet EPS $(r_1^*, \dots, r_{k-1}^*, r_{j_k}, \dots, r_{j_N})$ is rate-optimal, which gives the desired inequality by (c).

APPENDIX IV

PROOF OF LEMMA 3

Let $E = E_N(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t_i}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$. Then as in (8)

$$\begin{aligned} E &= E_{t_i}(r_{j_i}, \dots, r_{j_i}) + [q(r_{j_i})]^{t_i} E_{N-t_i}(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \\ &= E_{t_i}(r_{j_i}, \dots, r_{j_i}) + [q(r_{j_i})]^{t_i} E_{t_{i-1}}(r_{j_{i-1}}, \dots, r_{j_{i-1}}) + \\ &\quad [q(r_{j_i})]^{t_i} [q(r_{j_{i-1}})]^{t_{i-1}} E_{N-t_i-t_{i-1}}(\underbrace{r_{j_{i-2}}, \dots, r_{j_{i-2}}}_{t_{i-2}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \\ &= \sum_{l=0}^i E_{t_l}(r_{j_l}, \dots, r_{j_l}) \prod_{k=l+1}^i q(r_{j_k})^{t_k}. \end{aligned}$$

Similarly for $l = 0, \dots, i$

$$\begin{aligned} E_{t_l}(r_{j_l}, \dots, r_{j_l}) &= \sum_{k=0}^{t_l-1} E(r_{j_l}) q(r_{j_l})^k \\ &= T(r_{j_l}, t_l), \end{aligned}$$

which completes the proof.

APPENDIX V

PROOF OF LEMMA 4

Using Lemma 3, we have for $i \geq 0$

$$\begin{aligned} (A_i - B_i)q(r_{j_i})^{t-1} &= \frac{E_1(r_{j_i})}{1 - q(r_{j_i})} - T(r_{j_i}, t-1) - q(r_{j_i})^{t-1} \sum_{l=0}^{i-1} T(r_{j_l}, t_l) \prod_{k=l+1}^{i-1} q(r_{j_k})^{t_k} \\ &= \frac{E_1(r_{j_i})}{1 - q(r_{j_i})} - E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}), \end{aligned}$$

where for $i = 0$, the expression $E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ reads $E(\underbrace{r_{j_0}, \dots, r_{j_0}}_{t-1})$.

Thus, the sign of $\psi_{i,k}(t)$ is equal to that of

$$E_1(r_{j_i}) - E_1(r_k) - ((p(r_{j_i}) - p(r_k)) E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}))$$

or, equivalently, to that of

$$[E_1(r_{j_i}) - p(r_{j_i})E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})] - [E_1(r_k) - p(r_k)E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})],$$

which, by adding $E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ to the two terms between the brackets, is the sign of

$$E(\underbrace{r_{j_i}, r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) - E(\underbrace{r_k, r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}).$$

APPENDIX VI

PROOF OF LEMMA 5

$$\begin{aligned} A_i - B_i &= \frac{E_1(r_{j_i})}{1 - q(r_{j_i})} - E(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \\ &= \frac{1}{1 - q(r_{j_i})} (E_1(r_{j_i}) + q(r_{j_i})E(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})) - \\ &\quad \frac{1}{1 - q(r_{j_i})} E(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) \\ &= \frac{1}{1 - q(r_{j_i})} (E(r_{j_i}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) - \\ &\quad E(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})). \end{aligned} \tag{11}$$

Since $(\underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ is rate-optimal, the EPS $(r_{j_{i-1}}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ is rate-optimal over all EPSs whose first code rate is constrained to be $r_{j_{i-1}}$. Thus, by Proposition 1(b), (11) gives

$$\begin{aligned} A_i - B_i &\geq \frac{1}{1 - q(r_{j_i})} (E(r_{j_i}, \underbrace{r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) - \\ &\quad E(\underbrace{r_{j_{i-1}}, r_{j_{i-1}}, \dots, r_{j_{i-1}}}_{t_{i-1}}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})) \\ &> 0, \end{aligned}$$

where the last inequality is obtained from Lemma 4 using the assumption $\psi_{i-1, j_i}(t_{i-1} + 1) < 0$.

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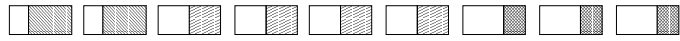
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Fig. 1: (a) System used in this paper: Fixed-length channel codewords with variable-length information blocks. (b) System used in [1] and [2]: Fixed-length information blocks with variable-length channel codewords. For both systems, the white areas correspond to information bits and the shaded areas to protection bits. The channel code rate is $1/4$ for the first two codewords, $1/2$ for the next four codewords, and $2/3$ for the last three codewords.

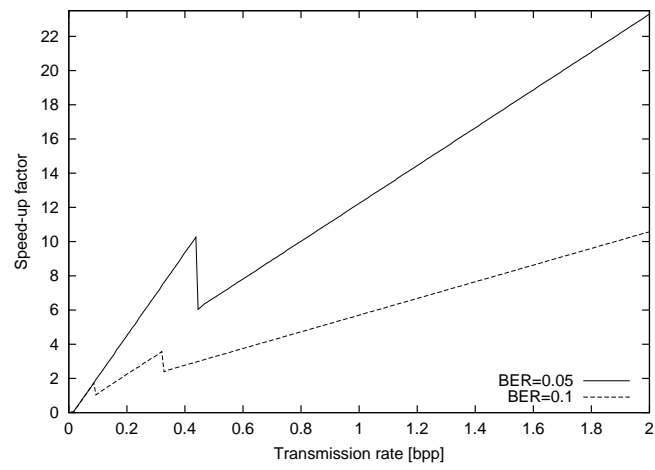
Fig. 2: Average speed-up factors of our algorithm over the algorithm of Proposition 2 for various target transmission rates. Results are given for two channel coders ((a) RCPT and (b) RCPC) and two BERs (0.05 and 0.1).



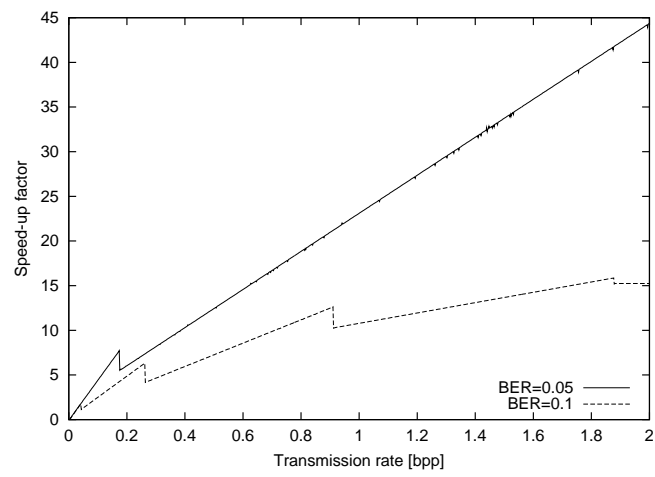
(a)



(b)



(a)



(b)