FAST ALGORITHM FOR OPTIMAL ERROR PROTECTION OF EMBEDDED WAVELET CODES

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Abstract - Embedded wavelet codes are very sensitive to channel noise because a single bit error can lead to an irreversible loss of synchronization between the encoder and the decoder. Sherwood and Zeger protected a zero-tree based embedded wavelet code sent through a memoryless noisy channel by using cyclic redundancy detection codes (CRC) and channel correction codes. Chande and Farvardin proposed an optimal joint source-channel allocation strategy for such systems. We show how to accelerate their algorithm without quality loss. For grey scale test images of size 512 $\times$ 512, our speedup factors ranged from 1.2 to 6 for total bit rates between 0.25 and 1.0 bits per pixel. Moreover, by using turbo codes as channel codes, we obtained competitive peak-signal-to-noise ratio (PSNR) results.

1. INTRODUCTION

The rate-distortion performance of zero-tree based wavelet coders (e.g., the SPIHT coder [6]) is competitive, both the encoding and the decoding are very fast, and the bit stream is embedded, allowing progressive transmission. The encoder uses a discrete wavelet transform to decompose the image into subbands. Coding of the wavelet coefficients exploits a tree-structured dependency across subbands at different scales and consists of a sequence of sorting and refinement passes. A single error in the bits sent in the sorting pass leads to a loss of synchronization between the encoder and the decoder. Errors in the refinement bits do not propagate, but still cause a big loss in PSNR. Zhao et al. [9] showed that one should stop decoding when the first error is detected because further decoding would make the reconstructed image worse. Sherwood and Zeger [7] protected the SPIHT code against noise in a memoryless binary symmetric channel (BSC) by concatenating CRC bits and rate-compatible punctured convolutional (RCPC) codes. Chande and Farvardin [2] and Zhao et al. [9, 10, 11] proposed optimal error protection algorithms for this system, which maximize the expected number of received source bits before an error occurs. In this paper, we find the optimal error protection with a faster strategy and give an implementation for turbo codes.

2. ALGORITHM FOR OPTIMAL BIT ALLOCATION

We assume that the source code is protected by a concatenation of CRC bits as an outer code and protection bits as an inner code. The bitstream is organized in packets of fixed length L. Packets are sent over a memoryless BSC. When the first error is detected, the decoding is stopped, and the image is reconstructed from the packets received until that point.

Let $\mathcal{R}$ be a set of m available channel rates $r_1 < \cdots < r_m$ with $p(r_1) < \cdots < p(r_m)$, where $p(r_j),\ j = 1, \ldots, m$, is the probability of an error in a packet protected by channel rate r_j. Given a target total number N of packets of L bits each to be sent, we want to find an *N-packet error protection scheme (EPS)* $(r_{k_1}, \ldots, r_{k_N})$ that assigns a channel rate r_{k_i} to each packet i such that the expected mean squared error (MSE) of the reconstructed image is minimum. For $i = 1, \ldots, N$, suppose that packet i is protected by channel rate r_{k_i}. Then for $i = 1, \ldots, N-1$, the number $P_i = p(r_{k_{i+1}}) \prod_{j=1}^{i}(1 - p(r_{k_j}))$ is the probability that no errors occur in the first i packets, with an error in the next one, and $P_N = \prod_{j=1}^{N}(1 - p(r_{k_j}))$ is the probability that no errors occur in the N packets. Because the image is reconstructed only from the packets received before the first error is detected, the expected MSE is $E_N[D](r_{k_1}, \ldots, r_{k_N}) = \sum_{i=0}^{N} P_i d_i$, where $P_0 = p(r_{k_1}),\ d_0$ is a constant, and for $i \geq 1,\ d_i$ is the MSE from the reconstruction using the first i (error-free) packets. It is equivalent, but computationally simpler, to maximize the expected number of source-encoder bits received in a total of N packets before an error occurs [2, 9], which is

$$E_N(r_{k_1}, \ldots, r_{k_N}) = \sum_{i=1}^{N} P_i \sum_{j=1}^{i} v(r_{k_j}), \tag{1}$$

where $v(r_{k_j})$ is the number of source bits in packet j protected by r_{k_j}. For every N an optimal equal error protection (EEP) scheme can easily be found. However, because in an embedded code the bits in the source code have decreasing importance, an unequal error protection (UEP) scheme where the channel rate is dynamically adjusted may be more efficient. Zhao et al. [10] proposed an algorithm (called two-rate UEP scheme) for finding the optimal N-packet error protection scheme, when only one channel rate change is allowed. The algorithm is time-consuming because it maximizes E_N for all possible N-tuples $(r_i, \ldots, r_i, r_j, \ldots, r_j),\ i, j \in \{1, \ldots, m\}$. In [11], a solution to the three-rate case is given. Chande and Farvardin [2] find a dynamic programming solution to the more general problem of maximizing (1) when an arbitrary N-packet protection scheme may be used. We now show how to accelerate this algorithm. For $r_j, r_k \in \mathcal{R}$ and any integer $i \geq 1$, let $q(r_j) = 1 - p(r_j),\ T(r_j, i) = \frac{E_1(r_j)(1-q(r_j)^i)}{1-q(r_j)}$, and $M(r_k, r_j) = \frac{E_1(r_k)(1-q(r_j)) - E_1(r_j)(1-q(r_k))}{(q(r_k)-q(r_j))(1-q(r_j))}$. Then we have:

Lemma 1 *Let $N \geq 1$. Then for any positive integers $t_0, \dots, t_i$, $N = t_0 + \cdots + t_i$, and channel rates $r_{j_0}, \dots, r_{j_i}$, $p(r_{j_k}) \neq 0, 0 \leq k \leq i$*

$$E_N(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t_i}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) = \sum_{l=0}^{i} T(r_{j_l}, t_l) \prod_{k=l+1}^{i} q(r_{j_k})^{t_k}.$$

Proof. The proof is obtained by successive applications of the equality

$$\begin{aligned} E_N(r_{k_1}, r_{k_2} \dots, r_{k_N}) &= (1 - p(r_{k_1}))(v(r_{k_1}) + E_{N-1}(r_{k_2}, \dots, r_{k_N})) \\ &= E_1(r_{k_1}) + q(r_{k_1}) E_{N-1}(r_{k_2}, \dots, r_{k_N}). \end{aligned}$$

Lemma 2 *If the $(N-1)$-packet EPS $(r_2^*, \dots, r_N^*)$ is optimal and if for all $r_{k_1} \neq r_1^*$, we have $E_N(r_1^*, r_2^*, \dots, r_N^*) > E_N(r_{k_1}, r_2^*, \dots, r_N^*)$, then the N-packet EPS $(r_1^*, \dots, r_N^*)$ is optimal.*

Proof. Let $r_{k_1}, r_{k_2}, \dots, r_{k_N} \in \mathcal{R}$. Then

$$\begin{aligned} E_N(r_1^*, r_2^*, \dots, r_N^*) &> E_N(r_{k_1}, r_2^*, \dots, r_N^*) \\ &= q(r_{k_1})(v(r_{k_1}) + E_{N-1}(r_2^*, \dots, r_N^*)) \\ &> E_N(r_{k_1}, r_{k_2}, \dots, r_{k_N}). \end{aligned}$$

Lemma 3 *If the N-packet EPS $(r_1^*, \dots, r_N^*)$ is optimal, then $r_1^* \leq \cdots \leq r_N^*$.*

Proof. Let $(r_1^*, \dots, r_N^*) = (\underbrace{r_{j_n}, \dots, r_{j_n}}_{t_n}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$ with $t_i \geq 1$ and $r_{j_i} \neq r_{j_{i+1}}$, $i \geq 0$. Then $E_{t_0+1}(r_{j_1}, r_{j_0}, \dots, r_{j_0}) > E_{t_0+1}(r_{j_0}, r_{j_0}, \dots, r_{j_0})$. Thus, $E_1(r_{j_0}) - E_1(r_{j_1}) < (p(r_{j_0}) - p(r_{j_1}))E_{t_0}(r_{j_0}, \dots, r_{j_0})$, which shows that $r_{j_1} < r_{j_0}$. Using similar techniques, one can show that $r_{j_{i+1}} < r_{j_i}$ for $i \geq 1$.

Proposition 1 *The optimal N-packet EPS is $(\underbrace{r_{j_n}, \dots, r_{j_n}}_{t_n}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0})$, where $r_{j_0}, \dots, r_{j_n}$ and $t_0, \dots, t_n$ are as follows. Let $r_{j_0} = \arg\max_{r_k} E_1(r_k)$. Set $i = 0$.*
1. Let $A_i = \frac{E_1(r_{j_i})}{1 - q(r_{j_i})}$ and $B_i = \sum_{l=0}^{i-1} T(r_{j_l}, t_l) \prod_{k=l+1}^{i-1} q(r_{j_k})^{t_k}$. Let $k \in \{1, \dots, m\}$ and $r_k < r_{j_i}$. If $a_{i,k} = \frac{\log \frac{M(r_k, r_{j_i})}{A_i - B_i}}{\log q(r_{j_i})} + 1$ exists and is finite, then set $t_{i,k} = \lfloor a_{i,k} \rfloor$. Otherwise, set $t_{i,k} = N - \sum_{p=0}^{i-1} t_p$. Let $t_i = \min_k t_{i,k}$.
2. If $N \leq \sum_{p=0}^{i} t_p$, set $r_{j_n} = r_{j_i}$, $t_n = t_i$ and stop. Otherwise, set $j_{i+1} = \arg\min_k t_{i,k}$ (we assume that j_{i+1} is unique), $i = i + 1$ and go to 1.

Proof. The proof is a consequence of Lemma 2, Lemma 3, and the fact that for all $0 \leq i \leq n$, if $1 \leq t \leq t_i$, then for all $r_k < r_{j_i}$

$$E(\underbrace{r_{j_i}, \dots, r_{j_i}}_{t}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}) > E(r_k, \underbrace{r_{j_i}, \dots, r_{j_i}}_{t-1}, \dots, \underbrace{r_{j_0}, \dots, r_{j_0}}_{t_0}), \tag{2}$$

Total rate (bpp)	$L = 2048$	$L = 1024$	$L = 512$
0.25	1.2	1.61	2.33
0.5	1.6	2.15	3.45
0.75	1.85	3.13	4.83
1.0	2.3	4	6

Table 1: Average (over BER = 0.05 and 0.1) speed-up factors of our algorithm over the one in [2] for various packet lengths L and target total rates.

and that for $1 \le i \le n$, (2) holds for $t = 1$ and $r_{j_i} < r_k \le r_{j_{i-1}}$. We start with $i = 0$. If $a_{i,k}$ exists and is finite, then $A_i - B_i > 0$. Thus $\psi_{i,k}(t) = (A_i - B_i)q(r_{j_i})^{t-1} - M(r_k, r_{j_i})$ is decreasing from $+\infty$ to $-M(r_k, r_{j_i})$. Since $\psi_{i,k}(1) > 0 = \psi_{i,k}(a_{i,k})$, we have $t_{i,k} > 0$, which ensures that $t_i > 0$. Let $1 \le t \le t_i$. Then $\psi_{i,k}(t) \ge \psi_{i,k}(t_i) \ge \psi_{i,k}(t_{i,k}) > \psi_{i,k}(a_{i,k}) = 0$. According to Lemma 1, the left-hand side of (2) is equal to $T(r_{j_i}, t) + q(r_{j_i})^t B_i$ and the right-hand side to $E_1(r_k) + q(r_k)(T(r_{j_i}, t-1) + q(r_{j_i})^{t-1} B_i)$. Thus $\psi_{i,k}(t) > 0$ gives (2) for $r_k < r_{j_i}$. Moreover, $\psi_{i,j_{i+1}}(t_{i,j_{i+1}} + 1) < \psi_{i,j_{i+1}}(a_{i,j_{i+1}}) = 0$, which gives $E(r_{j_{i+1}}, \underbrace{r_{j_i}, \ldots, r_{j_i}}_{t_i}) > E(\underbrace{r_{j_i}, \ldots, r_{j_i}}_{t_i+1})$. But $E(\underbrace{r_{j_i}, \ldots, r_{j_i}}_{t_i+1}) \ge E(r_k, \underbrace{r_{j_i}, \ldots, r_{j_i}}_{t_i})$ for $r_k \ne r_{j_{i+1}}$. Hence (2) is satisfied for $i = 1$, $t = 1$, and $r_{j_i} < r_k \le r_{j_{i-1}}$. Using the same approach, we can complete the proof for $i = 1, 2, \ldots, n$.

3. RESULTS

We protected the source bits of Fowler's implementation [3] of the SPIHT coder by concatenating CRC bits and rate compatible punctured turbo (RCPT) codes [4]. The turbo coder consisted of two identical recursive systematic convolutional encoders [5] with memory length 4 and generators (31, 27) octal. Puncturing was dynamically changed, yielding different code rates. The mother code was $20/60 = 1/3$, and the puncturing rate was 20. We used iterative maximum a posteriori decoding, which was stopped if no correct sequence was found after 20 iterations.

Table 1 compares the time complexity of our solution to that of Chande and Farvardin [2] for various packet lengths L and target total rates $R_T = NL/512^2$. In [2], the optimal solution is found by a repetitive use of Lemmas 2 and 3. Our algorithm determines t_i ahead of time, which avoids many unnecessary computations.

Figure 1 shows for two bit error rates, the difference in the expected number of received source bits between optimal UEP and optimal EEP for various target number of packets N, when $L = 2048$. For example, for $N \le 406$ and at BER = 0.1, EEP with rate 20/48 was optimal for $0 \le N \le 24$, EEP with rate

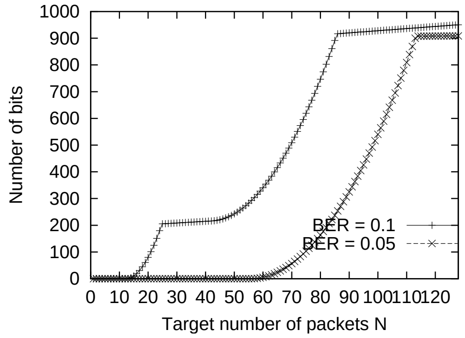

Figure 1: Difference between the expected number of received source bits for optimal UEP and optimal EEP for BER 0.05 and 0.1.

Total rate (bpp)	BER	Our system	[7]	[8]	[1]	[11]
0.25	0.01	32.54	31.91		32.56	
	0.05	31.27		31.52		
	0.1	29.75	28.5		29.13	
0.5	0.01	35.67	35.2		35.67	35.02
	0.05	34.26		34.53		
	0.1	32.64	31.23		32.03	31.3
1	0.01	38.72	38.03		38.78	38.07
	0.05	37.34				
	0.1	35.69	34.25		34.98	34.17

Table 2: PSNR for the 512 $\times$ 512 Lenna image.

20/50 was optimal for $25 \leq N \leq 86$, and EEP with rate 20/52 was optimal for $87 \leq N \leq 406$. Using the notation of Proposition 1, optimal UEP was given by $r_{j_0} = 20/48, t_0 = 11,\ r_{j_1} = 20/50, t_1 = 30$, and $r_{j_2} = 20/52, t_2 = 365$.

Table 2 shows PSNR results for the luminance part of the USC 512 $\times$ 512 Lenna image. The table compares our UEP results with $L = 2048$ to some of the best previously published ones. The side information needed to specify the optimal channel rates and the points of rate change was always less than 30 bits. Note that Banister et al. [1] used JPEG2000 coded images, which were protected by turbo codes. In [11], RCPC channel codes and a 3-rate dynamic UEP scheme were used. In [8], turbo codes were used. Here, in contrast to our work, the length of a packet was not fixed for all target rates. Note, however, that a greater packet size improves the PSNR performance but reduces the progressive ability.

5. CONCLUSION

We proposed a fast strategy for determining the optimal protection of embedded wavelet codes sent through a memoryless binary symmetric channel. Our algorithm improves the best previous solution. By using the SPIHT code as a source code and RCPT codes as channel codes, we obtained an embedded bitstream with state-of-the-art PSNR performance.

Acknowledgment. We thank Jim Fowler for providing his software, Minyi Zhao, Greg Sherwood, and Ken Zeger for sharing their experimental results.

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