PROGRESSIVE OPTIMAL ERROR PROTECTION OF EMBEDDED CODES

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I INTRODUCTION

The embedded wavelet image coders such as SPIHT [1] or JPEG 2000 [2] allow an efficient progressive transmission of digital images. However, the bitstreams generated by these coders are very sensitive to channel noise, and error protection is necessary to ensure acceptable reconstruction fidelity. One of the most successful protection systems for embedded wavelet coders was recently proposed by Sherwood and Zeger [3] who partitioned the bitstream of information bits into packets of fixed length and protected these packets by using a concatenation of a cyclic redundancy-check (CRC) coder for error detection and a rate-compatible punctured convolutional (RCPC) coder for error correction. A challenging task is to find an optimal error protection for such systems [4]; that is, an assignment of the available channel rates to the packets that minimizes the expected reconstruction error (measured, for example, by the peak-signal-to-noise ratio (PSNR)) subject to a total bit rate constraint. An alternative proposed in [4] is to maximize the expected number of error-free received source bits for this target total bit rate. Though suboptimal in the PSNR sense, this approach has two advantages. First, an optimal protection can be determined with a fast algorithm [4, 5]. Second, the solution is independent of both the source coder and the test image. Thus, the algorithm can also be implemented by the receiver, avoiding the need for sending side information, which would have to be strongly protected.

In progressive decoding, a very good performance is desirable not only at the target total rate, but also at all intermediate rates. In particular, a good image quality at the lowest rates is very useful in internet browsing because it allows the receiver to stop the transmission at an early stage whenever necessary. In this paper, we consider an error protection strategy that maximizes the average expected number of error-free received source bits over all intermediate rates. We prove many interesting properties of an optimal solution, present a linear-time algorithm (linear in the number of packets) for computing a solution, and compare its progressive performance (number of error-free received source bits and PSNR) to that of the standard strategy that is optimal at the target total rate [4] for the SPIHT and JPEG 2000 source coders with a binary symmetric channel and an RCPC channel coder.

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II OPTIMAL ERROR PROTECTION

We consider a bitstream generated by an embedded coder. We assume that the channel encoder is a concatenation of a CRC coder as an outer encoder and a rate compatible channel encoder as an inner encoder. The channel encoder transforms the bitstream into a sequence of channel codewords (packets) of fixed length L. Note that our system is slightly different from the original one [3, 4] in which the number of source bits in a packet is fixed, while the different channel codes have codewords of variable length. The channel codewords are sent through a memoryless binary symmetric channel. If the channel decoder detects an error, the transmission is stopped, and the image is reconstructed from the error-free source bits received until that point. Here we assume that the probability of an undetected error is zero. Let $\mathcal{R}$ be the set of the m channel code rates $r_1 < r_2 < \cdots < r_m$ with $p(r_1) < p(r_2) < \cdots < p(r_m) < 1$, where $p(r_k),\ k = 1, \ldots, m$, is the probability of a decoding error for a packet of length L protected by channel rate r_k. Suppose that we want to send N packets of L bits each. An N-*packet error protection scheme* (EPS) $(r_{k_1}, \ldots, r_{k_N}) \in \mathcal{R}^N$ protects packet $i,\ i = 1, \ldots, N$, by channel rate r_{k_i}. The expected number of error-free received source bits is

$$E_N(r_{k_1}, \ldots, r_{k_N}) = \sum_{i=1}^{N} P_i \sum_{j=1}^{i} v(r_{k_j}), \qquad (1)$$

where $v(r_{k_j})$ is the number of source bits in a packet protected by rate r_{k_j}, $P_i = \prod_{j=1}^{i}(1 - p(r_{k_j}))p(r_{k_{i+1}}),\ 1 \le i < N$, is the probability that no errors occur in the first i packets, with an error in the next one, and $P_N = \prod_{j=1}^{N}(1 - p(r_{k_j}))$ is the probability that all N packets are decoded without error. We say that an N-packet EPS is optimal if it maximizes $E_N(r_{k_1}, \ldots, r_{k_N})$ over $\mathcal{R}^N$. A direct computation of an optimal N-packet EPS is a difficult combinatorial optimization problem because the number of candidates is equal to m^N. But the problem can be solved efficiently as follows. It is noted in [4] that if the $(N-1)$-packet EPS $(r_2^*, \ldots, r_N^*)$ is optimal and if for all $r_{k_1} \in \mathcal{R}$, we have $E_N(r_1^*, r_2^*, \ldots, r_N^*) \ge E_N(r_{k_1}, r_2^*, \ldots, r_N^*)$, then the N-packet EPS $(r_1^*, \ldots, r_N^*)$ is optimal. Therefore an optimal N-packet EPS can be found by successively computing an optimal i-packet EPS $(r_{k_{N-i+1}}, \ldots, r_{k_N}),\ i = 1, \ldots, N$, where at each step only $r_{k_{N-i+1}}$ has to be determined. Moreover, as noted in [4] and proved in [5], if the N-packet EPS $(r_1^*, \cdots, r_N^*)$ is optimal, then $r_1^* \le \cdots \le r_N^*$, which increasingly limits the number of possible channel rates. An optimal N-packet EPS has therefore the form $(\underbrace{r_{j_n},\ldots,r_{j_n}}_{t_n},\ldots,\underbrace{r_{j_0},\cdots,r_{j_0}}_{t_0})$, where $n \geq 0$, $r_{j_0} > \cdots > r_{j_n} \in \mathcal{R}$, and $t_0,\ldots,t_n \geq 1$ with $\sum_{i=0}^{n} t_i = N$. In [5], the algorithm of Chande and Farvardin [4] is accelerated by finding ahead of time an explicit formula for t_i, $i = 0,\ldots,n$.

III PROGRESSIVE OPTIMAL PROTECTION

To measure the progressive performance of an N-packet EPS, one can compute the average expected number of error-free received source bits over all intermediate rates up to the target bit rate (this is a special case of the general performance measure introduced in [6]).

Definition 1 *An N-packet EPS is progressive optimal if it maximizes $L_N(R) = \sum_{n=1}^{N} E_n(r_{k_1},\ldots,r_{k_n})$ over all $R = (r_{k_1},\ldots,r_{k_n}) \in \mathcal{R}^N$.*

The following proposition gives an upper bound on the progressive performance of an N-packet EPS.

Proposition 1 *Let $R = (r_{k_1},\ldots,r_{k_N}) \in \mathcal{R}^N$ be an N-packet EPS. Let $(r_1^*,\ldots,r_N^*)$ be an optimal N-packet EPS. Then*

$$L_N(R) \leq \sum_{n=1}^{N} E_n(r^*_{N-n+1},\ldots,r^*_{N-1},r^*_N).$$

Proof. Let $r_{k_2},\ldots,r_{k_N} \in \mathcal{R}$. Then

$$E_N(r_1^*,r_2^*,\ldots,r_N^*) \geq E_N(r_1^*,r_{k_2},\ldots,r_{k_N}). \quad (2)$$

Using the equality [4]

$$E_N(r_{k_1},\ldots,r_{k_N}) = q(r_{k_1})(v(r_{k_1})+E_{N-1}(r_{k_2},\ldots,r_{k_N})) \quad (3)$$

where $q(r_{k_1}) = 1 - p(r_{k_1})$, equality (2) gives

$$E_{N-1}(r_2^*,\ldots,r_N^*) \geq E_{N-1}(r_{k_2},\ldots,r_{k_N}),$$

which shows that the $(N-1)$-packet EPS $(r_2^*,\ldots,r_N^*)$ is optimal. Similarly, for $n = N-2,\ldots,1$, the n-packet EPS $(r^*_{N-n+1},\ldots,r^*_N)$ is optimal. □

Let $R = (r_{k_1},\ldots,r_{k_N})$ be an N-packet EPS and let $(r_1^*,\ldots,r_N^*)$ be an optimal N-packet EPS. We denote by $\Delta_n(R)$ the difference in the expected number of error-free received source bits after the first n packets have been sent between an optimal n-packet EPS and R, that is,

$$\Delta_n(R) = E_n(r^*_{N-n+1},\ldots,r^*_{N-1},r^*_N) - E_n(r_{k_1},\ldots,r_{k_n}).$$

It is interesting to ask if there exists an N-packet EPS $R = (r_{k_1},\ldots,r_{k_N})$ such that $\sum_{n=1}^{N}\Delta_n(R) = 0$. In this case, R would be progressive optimal, and the n-packet EPS $(r_{k_1},\ldots,r_{k_n})$ would be optimal for each $n \in \{1,\ldots,N\}$. The following proposition answers this question.

Proposition 2 *Let R be an N-packet EPS and let $r_{j_0} = \arg\max_{r_k\in\mathcal{R}} E_1(r_k)$. Then the following three statements are equivalent:*
(i) $\sum_{n=1}^{N}\Delta_n(R) = 0$.
(ii) For all positive integers $t \leq N$ and for all $r_k \in \mathcal{R}/\{r_{j_0}\}$, we have

$$\frac{E_1(r_{j_0})}{p(r_{j_0})}(1-p(r_{j_0}))^{t-1} - \frac{E_1(r_k)p(r_{j_0}) - E_1(r_{j_0})p(r_k)}{(p(r_{j_0}) - p(r_k))p(r_{j_0})} > 0.$$

(iii) $R = (r_{j_0},\ldots,r_{j_0})$.

Proof. The proof follows from Proposition 1 in [5]. □

We now consider the problem of computing a progressive optimal N-packet EPS. The following proposition shows that the complexity of the problem can be reduced to $O(N)$.

Proposition 3 *If the N-packet EPS $(r_1^p,\ldots,r_N^p)$ is progressive optimal, then the $(N-1)$-packet EPS $(r_2^p,\ldots,r_N^p)$ is also progressive optimal.*

Proof. Suppose that $R^p = (r_1^p,\ldots,r_N^p)$ is progressive optimal. Let $r_{k_1},\ldots,r_{k_{N-1}}$ be arbitrary rates in $\mathcal{R}$ and let $R = (r_1^p, r_{k_1},\ldots,r_{k_{N-1}})$. Then $L_N(R^p) \geq L_N(R)$. But

$$L_N(R^p) = NE_1(r_1^p) + (1-p(r_1^p))L_{N-1}(r_2^p,\ldots,r_N^p)$$

and

$$L_N(R) = NE_1(r_1^p) + (1-p(r_1^p))L_{N-1}(r_{k_1},\ldots,r_{k_{N-1}}).$$

Thus, $L_{N-1}(r_2^p,\ldots,r_N^p) \geq L_{N-1}(r_{k_1},\ldots,r_{k_{N-1}})$. □

A straightforward consequence of Proposition 3 is that for all $1 \leq i \leq N-1$ the $(N-i)$-packet EPS $(r_{i+1}^p,\ldots,r_N^p)$ is progressive optimal. We now show that the last packet of an optimal progressive EPS should have the same or a better protection than the other packets. This result reduces the number of channel rates that have to be considered for packet $i < N$.

Proposition 4 *Let $R^p = (r_1^p,\ldots,r_N^p)$ be an N-packet progressive optimal EPS. Then $r_N^p \geq r_k^p$ for all $k = 1,\ldots,N$.*

Proof. By Proposition 3, the 1-packet EPS (r_N^p) is progressive optimal. Thus, $E(r_N^p) \geq E(r_i)$ for all $i = 1,\ldots,m$. Suppose now that there exists k, $1 \leq k < N$, such that $r_k^p > r_N^p$. Then, by setting $N-k+1 = n$ and $(r_k^p,\ldots,r_N^p) = R$, we have

$$\begin{aligned}
L_n(R) &= nE(r_k^p) + (1-p(r_k^p))L_{N-k}(r_{k+1}^p,\ldots,r_N^p)\\
&\leq nE(r_N^p) + (1-p(r_k^p))L_{N-k}(r_{k+1}^p,\ldots,r_N^p)\\
&< nE(r_N^p) + (1-p(r_N^p))L_{N-k}(r_{k+1}^p,\ldots,r_N^p)\\
&= L_n(r_N^p, r_{k+1}^p,\ldots,r_N^p),
\end{aligned}$$

which contradicts the fact that the n-packet EPS R is progressive optimal as required by Proposition 3. □

The following algorithm efficiently finds a progressive optimal N-packet EPS by exploiting the two previous propositions.

Proposition 5 *Let N be a positive integer. A progressive optimal N-packet EPS $(r_1^p,\ldots,r_N^p)$ can be computed as follows.*
1. Set $i = 1$ and $j_1 = \arg\max_{k=1,\ldots,m} E_1(r_k)$.
2. If $i = N$, then set $(r_1^p,\ldots,r_N^p) = (r_{j_i},\ldots,r_{j_1})$ and

stop.

3. Set $i = i + 1$ *and* $j_i = \arg\max_{k,\, r_k \le r_{j_1}} L_{i,k}$ *with*

$$L_{i,k} = iE_1(r_k) + (1 - p(r_k))L_{i-1}(r_{j_{i-1}}, \ldots, r_{j_1}).$$

4. Go to Step 2.

Proof. The proof follows from Proposition 3, Proposition 4, and the equality $L_{i,k} = L_i(r_k, r_{j_{i-1}}, \ldots, r_{j_1})$. $\square$

IV RESULTS

In this section, we compare the performance of progressive optimal EPS to that of optimal EPS. In all experiments, we used a binary symmetric channel. The test image was the 8 bits per pixel (bpp) 512×512 Lenna. Note, however, that the optimal error protection solutions yielded by the two algorithms are independent of the test image.

The channel coder was a concatenation of the CRC coder of [3] and an RCPC coder from [7] with mother code memory length 6, generator polynomial $(147, 163, 135, 135)$, and rate $\frac{1}{4}$. The puncturing period was $p = 8$. The decoding was based on a tree-trellis search technique [8]. If the path selected by the Viterbi decoder was not declared error-free by the CRC test, the decoder selected the next best path. This procedure was repeated until the CRC test was passed or until 100 paths were considered. The total packet length was $L = 512$ bits consisting of information bits, 16 CRC detection bits, 6 bits for setting the convolutional encoder into a state of all zeroes, and protection bits. We provide results for $N = 512$ packets, which corresponds to a target total rate of 1 bpp. At channel bit error rate BER $= 0.1$, progressive optimal 512-packet EPS selected channel rate $\frac{8}{25}$ for protecting the first 237 packets, channel rate $\frac{8}{24}$ for the next 230 packets, channel rate $\frac{8}{22}$ for the next 44 packets, and channel rate $\frac{8}{20}$ for the remaining packet, while the optimal 512-packet EPS selected channel rate $\frac{8}{27}$ for protecting the first 44 packets, channel rate $\frac{8}{25}$ for the next 333 packets, channel rate $\frac{8}{24}$ for the next 113 packets, channel rate $\frac{8}{22}$ for the next 19 packets, and channel rate $\frac{8}{20}$ for the remaining three packets. At BER $= 0.05$, progressive optimal EPS selected channel rate $\frac{8}{18}$ for the first 328 packets, channel rate $\frac{8}{16}$ for the next 176 packets, and channel rate $\frac{8}{14}$ for the remaining packets, while the optimal 512-packet EPS selected channel rate $\frac{8}{18}$ for protecting the first 424 packets, channel rate $\frac{8}{16}$ for the next 86 packets, and channel rate $\frac{8}{14}$ for the remaining packets. Figure 1 shows the variations of $\Delta_n(R)$ as a function of n when the BER was 0.1. The experiment shows that the performance of an optimal N-packet EPS can be far from its upper bound at many intermediate rates. Figure 2 shows the difference in the expected number of received source bits between progressive optimal EPS and optimal 512-packet EPS for BERs 0.05 and 0.1. The experimental results show that the progressive optimal strategy had an equal or better performance at most of the intermediate rates and a slightly worse performance at rates close to the target rate. Figure 3 and 4 show that the same result holds if we now consider the PSNR performance. Here the source code was generated with the SPIHT coder. Finally, Figure 5 and Figure 6 show the results of similar experiments when the source coder was JPEG 2000.

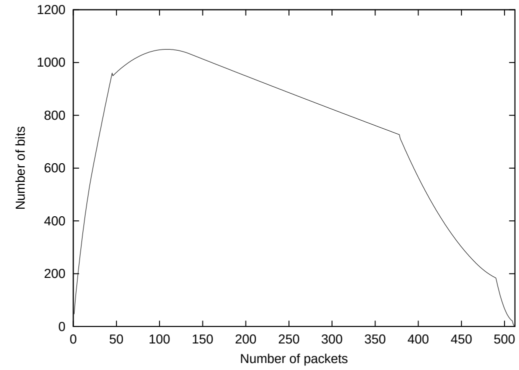

Fig. 1. Difference in the expected number of received source bits as a function of the number n of packets sent between n-packet optimal EPS and an EPS that is optimal for target rate 1.0 bpp. The BER was equal to 0.1.

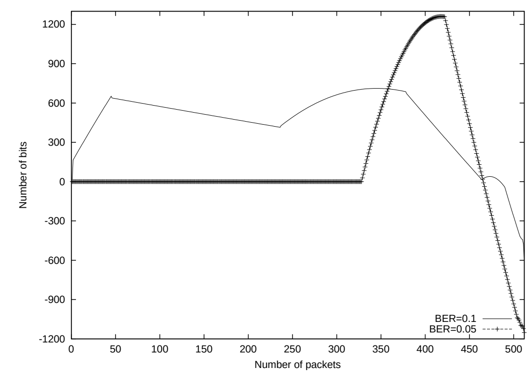

Fig. 2. Difference in the expected number of received SPIHT source bits between progressive optimal EPS and an optimal 1 bpp (512-packet) EPS for BER 0.05 and 0.1.

V CONCLUSION

The error protection scheme that maximizes the average expected number of error-free received source bits over all intermediate rates can be computed in linear time. Moreover, compared to the scheme that maximizes the expected number of error-free received source bits at a target rate [4], this scheme had a slightly worse performance (expected number of received bits and expected PSNR) at a few rates close to

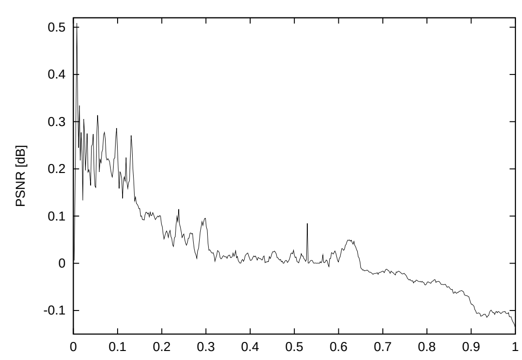

Fig. 3. Difference in expected PSNR between progressive optimal EPS and an optimal 1 bpp EPS for BER 0.1.

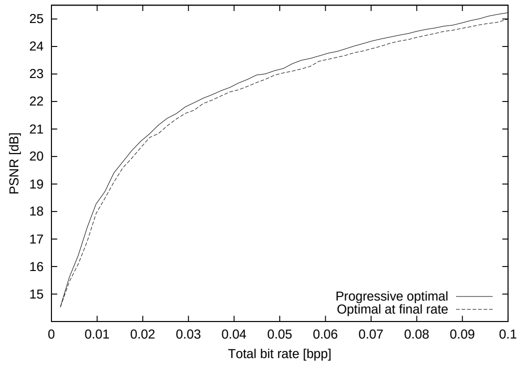

Fig. 4. Expected PSNR for progressive optimal EPS and an optimal 1 bpp EPS for BER 0.1. Results are shown up to total rate 0.1 bpp.

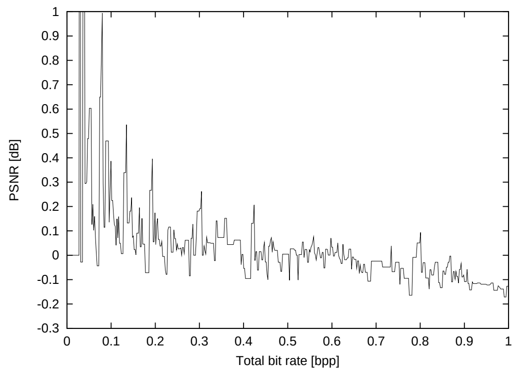

Fig. 5. JPEG 2000: Difference in expected PSNR between progressive optimal EPS and an optimal 1 bpp EPS for BER 0.1. For clarity, the PSNR axis was truncated at 1 dB.

the target total rate and a better performance at most of the intermediate rates. This makes the proposed scheme suitable for progressive transmission.

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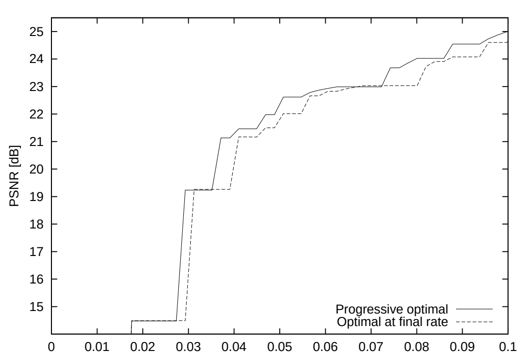

Fig. 6. JPEG 2000: Expected PSNR between progressive optimal EPS and an optimal 1 bpp EPS for BER 0.1.

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Sadržaj: Pošto su koderi za kompresiju slike, koji koriste vejvlet transformaciju i proizvode ugradjeni kod, veoma osetljivi na uticaj kanalnog šuma, neophodna je upotreba zaštitnog kodovanja. Sistemi za zaštitu od grešaka koji nude optimalnu raspodelu izvora i kanala samo za jedan ciljni broj prenetih paketa mogu da pokazuju nezadovoljavajuće performanse u toku prenosa, (na manjem broju prenetih paketa), što predstavlja veliki nedostatak kod progresivnog dekodovanja. Mi razmatramo sistem koji optimizira srednji broj korektno primljenih bita za sve intermedijalne kodne količnike, dokazujemo neke interesantne osobine optimalnog rešenja i predlažemo brz algoritam za njegovo nalaženje. Takodje uporedjujemo karakteristike ovog sistema sa standardnim, optimalnim samo za ciljni kodni količnik. Iz eksperimentalnih rezultata za binarni, simetrični kanal može se videti da predloženo rešenje pokazuje zanemarljivo gore rezultate na ciljnim kodnim količnicima i bolje performanse na većini intermedijalnih kodnih količnika.

PROGRESIVNA OPTIMALNA ZAŠTITA UGRADJENIH KODOVA, Vladimir Stanković, Raouf Hamzaoui.