FUZZY CLUSTERING FOR FRACTAL IMAGE COMPRESSION WITH APPLICATIONS TO DIGITAL ANGIOGRAPHY

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Abstract: Fractal image compression is a recent technique for lossy image coding which is based on finding local self-similarities in digital images. The encoding step in this procedure is computationally expensive. A large number of sequential searches through a list of domains (portions of the image) are carried out while trying to find a best match for another image portion called range. One way to overcome this difficulty is to use an adaptive clustering scheme for the domains and ranges so that only those domains are tested which are in the same cluster as the range. In previous attempts the membership of a domain or range for a cluster was unique (crisp). In this paper we propose a fuzzy version of clustering aiming at improving the reconstructed image quality while maintaining the advantage of the complexity reduction. Medical imagery seems to be a suitable class of images for fractal compression because of the inherent fractal nature of organic matter. In particular, when considering only a particular class of images such as digital angiography we can take advantage of this coherency by precomputing cluster centers from a set of training images. We also discuss the important issue of image quality for medical applications.

1 Introduction

With the ever increasing demand for images, sound, video sequences, computer animations and volume visualization, data compression remains a critical issue regarding the cost of data storage and transmission times. While JPEG currently provides the industry standard for still image compression there is ongoing research in alternative methods. Fractal image compression is one of them.

Basically, a fractal code consists of three ingredients: a partitioning of the image region into portions R_k, called *ranges,* an equal number of other image regions D_k, called *domains* (which may overlap), and for each domain-range pair two *transformations*, a geometric one, $u_k : D_k \to R_k$, which maps the domain to the range, and an affine transformation, v_k, that adjusts the intensity values in the domain to those in the range. The collection of transformations may act on an arbitrary image, g, producing an output image, Tg, which is like a collage of modified copies of the domains of the image g. The iteration of the image operator T is the decoding step, i.e., it yields a sequence of images which converges to an approximation of the encoded image.

The time consuming part of the encoding step is the search for an appropriate domain for each range. The number of possible domains that theoretically may serve as candidates is prohibitively large. For example, the number of arbitrarily sized square subregions in an image of size n by n pixel is of order $O(n^3)$. Thus, one must impose certain restrictions in the specification of the allowable domains. In a simple implementation one might consider as domains, e.g., only sub-squares of a limited number of sizes and positions. This defines the so-called *domain pool.* Now for each range in the partition of the original image all elements of the domain pool are inspected: for a given range R_k and a domain D the transformations u_k and v_k are constructed such that when the domain image portion is mapped into the range the result $v_k f u_k^{-1}(x)$ for $x \in R_k$ matches the original image f as much as possible. This step (called collage coding) uses the well-known least squares method. From all domains in the pool we select the best one, i.e., the domain D_k that yields the best least squares approximation of the original image in the range. In other words, fractal image coding consists in approximating the image as a collage of transformed pieces of itself, which can be viewed as a collection of self-similarity properties. The better the collage fits the given image the higher the fidelity of the resulting decoded image.

In this article we cannot explain many more details and variations of fractal image compression. For introductory texts or reviews see [1, 3, 4, 8]. For a bibliographic survey of the field see the paper [14].

Now it is common understanding that fractal image compression can deliver a performance in terms of a rate-distortion curve that compares favorably to, e.g., JPEG and which has a definite edge at very high compression ratios. Moreover, fractal image compression provides fast decoding and the code is resolution independent. However, the method suffers from long encoding times, which is its major deficiency. During the encoding a large pool of image subsets, called domains, has to be searched repeatedly many times, which by far dominates all other computations in the encoding process. If the number of domains in the pool is N, then the time spent for each search is *linear* in N, $O(N)$. Several methods have been devised to reduce the time complexity of the searching. They all work by some way of *feature extraction.* One can consider a finite set of discrete features, which are chosen either a priori fixed or adaptively, i.e., depending on the image. Alternatively, it has been proposed to use continuous features, either a single one or several simultaneously yielding feature vectors. For a survey of these complexity reduction methods see [15]. In this paper we consider only discrete features leading to *adaptive clustering* methods.

In the more common classification methods domains are grouped independently and online into predefined classes of domains [7, 6, 5]. For a given range only the class of the range is searched for a matching domain. In adaptive clustering methods domains and ranges are grouped around cluster centers which, however, depend on the image and, thus, on the set of ranges and domains [9, 10]. As in the classification method, only the class of the given range is searched for a matching domain. In this paper we propose a fuzzy version of clustering [2, 16] with the goal in mind that this approach improves the reconstructed image quality while maintaining the advantage of the complexity reduction. The idea is to consider a variable degree of membership of a range to different classes or clusters and then to search in those classes or clusters with the largest sympathy number.

We applied the fractal encoding to medical images, namely digital angiograms in cardiology. One of the major objectives in angiography is the visualization of anatomical details of a vessel segment of interest. Safety and success of diagnostic and interventional angiography depend on the proper visualization of all potentially relevant details. In case of the moving coronary vessel system this is done by recording an X-ray movie consuming 250 MBytes per patient. This combination of high data flow and uncompromising image quality requirements makes digital coding of the heart very difficult. Medical imagery seems to be a suitable class of images for fractal compression because of the inherent fractal nature of organic matter. In particular, when considering only a particular class of images such as digital angiography we can take advantage of this coherency by precomputing cluster centers from a set of training images.

The remainder of this article is organized as follows. In section 2 we present the fuzzy clustering approach to complexity reduction of fractal image compression. In section 3 we discuss the applications to digital angiography and show some examples. The last section provides a summary and points to future work in this research area.

2 Fuzzy clustering for fractal image compression

To set the stage for the fuzzy clustering we need some preparations.

We consider a point $z \in \mathbf{R}^d$ representing d pixel values in a given range. Let us assume that the domains from the domain pool have been properly filtered and subsampled, thus, yielding a set of N vectors $x^{(1)}, \ldots, x^{(N)} \in \mathbf{R}^d$ (often called codebook blocks). We let $E(x^{(i)}, z)$ denote the smallest possible least squares error of an approximation of the range data z by an affine transformation of the domain data $x^{(i)}$. In terms of a formula, this is $E(x^{(i)}, z) = \min_{a,b \in \mathbf{R}} ||z - (ae + bx^{(i)})||^2$, where $e = \frac{1}{\sqrt{d}}(1, \ldots, 1) \in \mathbf{R}^d$ is just a unit length vector with equal components. Computing the optimal a, b and the error $E(x^{(i)}, z)$ is a costly procedure (least squares optimization), which we have to perform for all of the domain vectors $x^{(1)}, \ldots, x^{(N)}$ in order to arrive at the minimum error, given by $\min_{1 \le i \le N} E(x^{(i)}, z)$. This staggered minimization problem needs to be solved for many query points z in the encoding process (i.e., for all ranges).

According to a central result (see [13]) we have that this minimization of the least squares errors $E(x^{(i)}, z)$ for $i = 1, \ldots, N$ is equivalent to the minimization of the Euclidean distance between certain multi-dimensional keys of the ranges and domains. These keys are simply the image intensity vectors $x^{(i)}$ and z with their DC components removed and normalized to unit length. In order to speed up the nearest neighbor calculation clustering of the keys can be employed. Such clusters can be derived using one of a variety of methods, such as the k-means algorithm, the self-organizing map of Kohonen, or the FCM (fuzzy c-means) to name just a few.

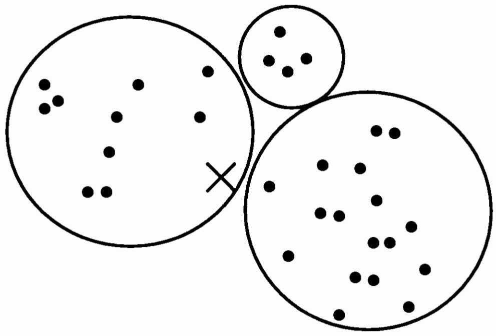

Figure 1: Three clusters of domain keys (dots) and a range key (cross). The nearest domain is not found in the cluster to which the range belongs. Fuzzy cluster membership can retrieve the closest neighbor.

After the clusters have been determined the collage coding may start. Each range will be associated with a certain cluster of domains according to the minimal distance between the range key and all cluster centers. However, the nearest neighbor of the range key does not necessarily lie in the closest cluster, see figure 1. Therefore it is of advantage to consider a fuzzy cluster membership of ranges. For example, the membership can be scaled according to distance to cluster centers. Then, in order to find a matching domain, the m clusters with the largest membership indicator can be searched. Here m is a parameter of the algorithm which can be user controlled. A large value of m will yield a smaller collage error and thus better image quality, however, at the expense of increased computing time. Corresponding computer studies are currently being prepared. We will evaluate the trade offs between the parameter settings, i.e., total number of classes and the number m of classes searched versus the computing time and the achieved image quality.

3 Digital angiography and fractal image compression

Cardiac angiography is used to display the coronary arteries supplying the heart muscle with blood. A contracted or closed vessel can cause a heart attack because of the reduced blood flow to certain regions of the muscle. The visualization of this vessel system, the so called coronary artery tree, is done by recording of X-ray movies under selective injection of contrast fluid via a catheter. Since over fifteen years, this method is state of the art for the diagnostic of the morphology and motion of the heart. Intervention techniques like the dilation of a contracted vessel segment by a balloon are widely used [11]. Cinefilm is still the gold standard for image sequence archiving in angiography because of its perfect visual resolution, longterm stability and global exchange compatibility. Although modern digital cathlab imaging equipment can offer important online visualization improvements like looped playback or realtime edge enhancement, a digital archive for the heart sequences is presently not available. One patient consumes about 250 MB of uncompressed data loading the lab's archive with gigabytes, which has to be transferred in a few minutes to be comparable to cinefilm handling [12]. Therefore strong data reduction is required.

Safety and success of diagnostic and interventional angiography depend on the proper visualization of all potentially relevant medical details. Image quality can be described as the usability of the acquired image sequences for all possible clinical evaluation needs. Most cases can be reported on the digital display immediately after the investigation, but waiting for the development of the higher resolving film can offer additional information in some cases: the visual detection of diagnostically significant small structures (e.g. vessel dissections, thrombotic lesions) potentially needs all resolution available. Every digital successor of the cinefilm consequently has to guarantee, that all medically relevant image details are fully preserved. Therefore, most cardiologists demand for an image archive with film-like resolution and non-lossy image compression. This combination of high data flow and uncompromising image quality requirements makes the digital image coding of the heart very difficult. A widely accepted compromise between maximum image quality and minimum data size seems to be difficult to find.

An efficient data reduction strategy for that particular class of images should use as much knowledge about the image content as possible. The major property of that class is the high degree of similarity of different vessels in one frame, different frames of a sequence and different patients. Therefore, fractal image compression is one of the most interesting approaches providing higher compression ratios at higher degrees of quality as compared to all-purpose-compression like JPEG. High decoding speed is important for quick distribution and replay in hospital and elsewere. The comparably slow encoding process has to be done once only and can be improved by clustering as described.

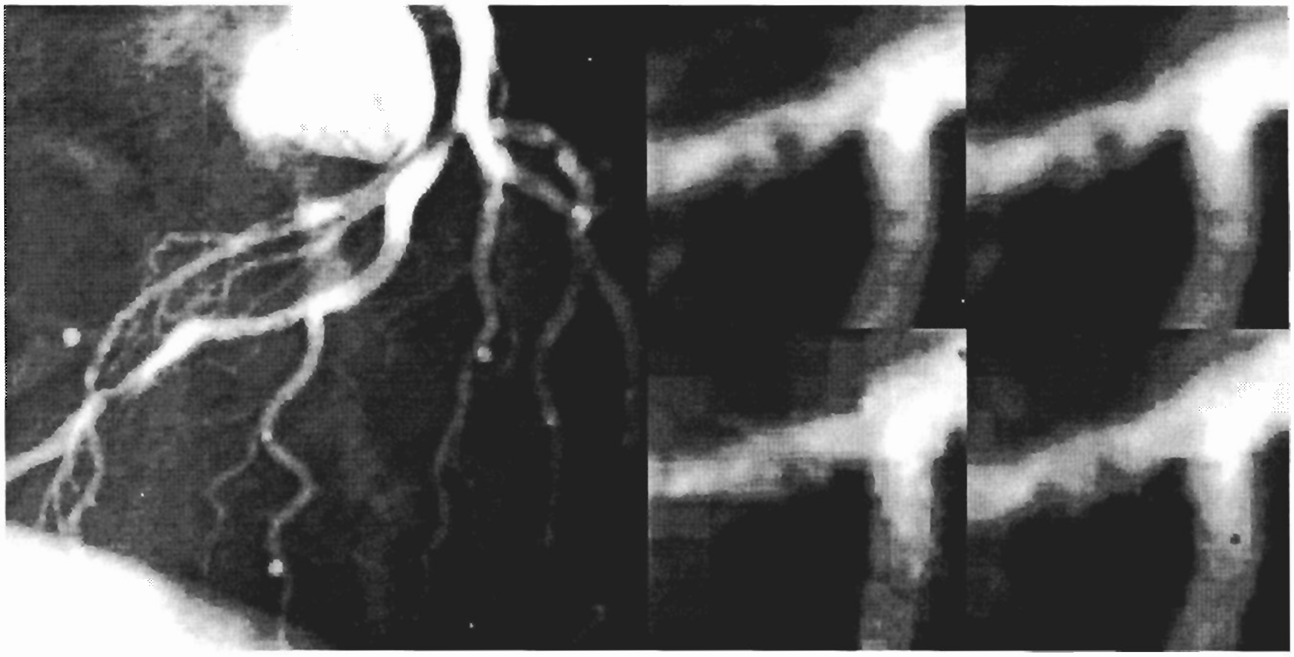

Figure 2:Angiogram with magnifications of the worst case object: The original image (upper left) and lossy images with compression rate/PSNR of (clockwise) 20:1/43db , 60:1/38db and 120:1/35db.

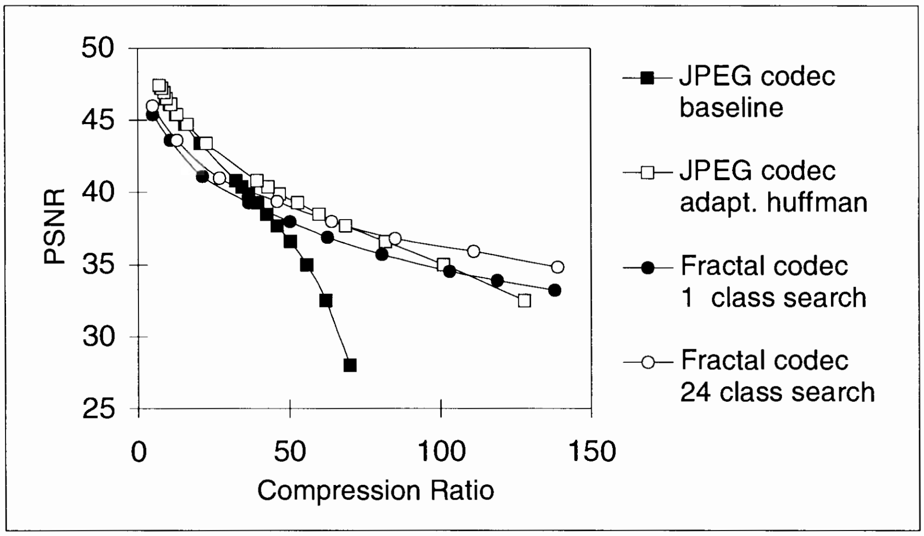

Figure 3: Performance of JPEG variants and the adaptive quadtree method on 512 × 512, 8 bit.

In a pilot study we used the fractal codec of Yuval Fisher [5] to compress angiograms. The sources of this implementation are free[1], various options allow an 'easy playing' with the tuning parameters of the algorithm. In Figure 3, the rate-distortion-performance of this codec is referenced to the JPEG algorithm and its non-standard extension. For higher compression rates, the quality of the fractal codec becomes better than JPEG in terms of the commonly used peak-signal-to-noise-ratio. Figure 2 shows the used angiogram and illustrates the image quality around the most critical object, a dark drop in the vessel. At 20:1 compression, the drop is clearly visualized. Although the overall image quality of the 60:1 compression satisfies the average observer, the thrombotic lesion in the vessel is poorly detectable. It is clear, that the PSNR measured over the whole frame cannot describe the local image quality in that point of medical interest.

[1]available via http://inls.ucsd.edu/y/Fractals

4 Summary and conclusion

Fractal image compression is a lossy image coding technique using the self-similarity of the image contents. A fast fractal image compression method has been proposed, which uses different membership values for comparison of image portions (ranges) with sets of other portions (domains) of the same image. The main disadvantage of the fractal compression is the encoding complexity. While domain clustering yields a decrease of computational effort due to less domain-range comparisons, the sympathy values introduced by fuzzy clustering gives the next probable clusters, when no sufficient similarity could be calculated. This extension lowers the complexity problem while potentially increasing the achievable image quality. Further experiments have to be done on the field of nearest neighbour search and fuzzy clustering to demonstrate the approach outlined. We found that in fact digital angiograms transform excellent to a fractal collage as expected from the high degree of self-similarity in the image. Generally, fractal image compression seems to be very promising for medical imaging, because of the inherent fractal nature of organic matter. In medicine however, every lossy compression technique has to guarantee, that all medically relevant details are fully preserved. Unfortunatly, technical quality measures do not reflect the visual similarity of medical images from the physicians point of view. Thus, linguistic variables could help to adapt the local encoding quality based on the medical relevance of different image contents. The development of a fuzzy approach to a medical image quality measure is a future perspective.

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