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Rate-based versus distortion-based optimal error protection of embedded codes

Raouf Hamzaoui, Vladimir Stanković, Zixiang Xiong

Abstract

We consider a joint source-channel coding system that protects an embedded bitstream using a finite family of channel codes with error detection and error correction capability. The performance of this system may be measured by the expected distortion or by the expected number of correctly decoded source bits. Whereas a rate-based optimal solution can be found in linear time, the computation of a distortion-based optimal solution is prohibitive. Under the assumption of the convexity of the operational distortion-rate function of the source coder, we give a lower bound on the expected distortion of a distortion-based optimal solution that depends only on a rate-based optimal solution. Then, we conjecture that a distortion-based optimal solution uses the same number or fewer information bits than a rate-based optimal solution. Finally, we propose a local search algorithm that starts from a rate-based optimal solution and converges in linear time to a local minimum of the expected distortion. Experimental results for a binary symmetric channel show that our local search algorithm is near optimal, whereas its complexity is much lower than that of the previous best solution.

I. Introduction

One of the most efficient systems for the progressive transmission of images over memoryless noisy channels without feedback was proposed by Sherwood and Zeger [1]. The basic idea is to use an embedded wavelet coder for source coding and a concatenation of an outer cyclic redundancy check (CRC) coder and an inner rate-compatible punctured convolutional (RCPC) coder for channel coding. Error propagation is avoided by stopping the decoding when the first error is detected. In the original setting [1], the information bits were organized in blocks of fixed length, each of which was mapped to a channel codeword of a variable length. However, it is more convenient to fix the size of the channel codewords and to allow the blocks of information bits to have a variable length [2, 3] (see Figure 1).

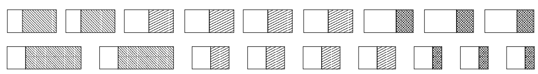

Fig. 1. (Top) Fixed-length channel codewords with variable-length information blocks. (Bottom) Fixed-length information blocks with variable-length channel codewords. The white areas correspond to information bits and the shaded areas to protection bits.

A challenging problem for these systems is to find an allocation of the total transmission rate between the source coder and the channel coder that minimizes the expected distortion. For the fixed-length information block case, Lu, Nosratinia, and Aazhang [4] show that under some assumptions, the channel code rates of an optimal solution should be nondecreasing in the information block number, which significantly reduces the complexity of an exhaustive search. Also in the fixed-length information block case, Chande and Farvardin [5] provide a dynamic programming solution to the optimization problem and report an $O(R^2)$ time complexity, where R is the target transmission rate. In the fixed-length channel codeword case, however, no fast exact solution is known. To the best of our knowledge, the best approximation to an optimal

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solution was given in [2]. It is based on a Viterbi algorithm and has $O(R^2)$ complexity. However, this result is guaranteed only for channel code rates that are a subset of $\{\frac{p}{q}, \frac{p+1}{q}, \ldots, \frac{q-1}{q}\}$, where p and q are positive integers with $p < q$. For many channel codes, including rate compatible punctured codes, the worst case time complexity is exponential in the total transmission rate.

An alternative to minimizing the expected distortion is to maximize the expected number of correctly decoded source bits (this approach was introduced by Chande, Jafarkhani, and Farvardin in [6] for a similar system that uses a feedback channel). Though suboptimal in the distortion sense, the rate-based optimization has two main advantages: an optimal solution can be computed with a linear-time algorithm (see [5] for fixed-length information blocks and [3] for fixed-length channel codewords), and it is independent of both the source coder performance and the image; thus, it can be determined by the receiver, which avoids the need for side information. In [5], experimental results show that the solutions to the two optimization problems have a similar performance for the SPIHT coder and an RCPC channel coder (the loss in average peak-signal-to-noise ratio (PSNR) was less than 0.2 dB for the 512 × 512 gray-scale Lenna image). Hedayat and Nosratinia [7] analytically confirm these results under many assumptions, including an i.i.d. Gaussian source and a perfect progressive source coder that achieves the rate-distortion function.

In this paper, we study the rate-based and the distortion-based optimization problems in the context of fixed-length channel codewords. In Section III, we provide a theoretical upper bound on the difference in expected distortion between the solutions of the two optimization problems under the assumption of the convexity of the operational distortion-rate function of the source coder. We also conjecture that the total number of information bits of a distortion-based optimal solution is equal to or smaller than that of a rate-based optimal solution. This allows us to reduce the complexity of the distortion-based minimization by eliminating the solutions that have more information bits than a rate-based optimal solution. In Section IV, we propose a fast local search algorithm that starts from a rate-based optimal solution and tries to minimize the expected distortion by successively reducing the number of information bits (or equivalently by increasing the number of protection bits). Section V presents numerical results for SPIHT [8] and JPEG2000 [9]. We show in particular that the local search algorithm yields a comparable solution to that obtained with the Viterbi algorithm of [2], but at a much lower complexity.

II. Optimization Criteria

We consider a source-channel coding system that uses an embedded source coder and a finite family $C_1, \ldots, C_m$ of channel codes with error detection and error correction capability. We recall that a source coder is called embedded if for any integers B_1 and B_2 with $B_1 < B_2$, the output bitstream of length B_1 is a prefix of the output bitstream of length B_2. Given a transmission bit budget B and a channel packet size L, the channel encoder transforms $N = \lfloor B/L \rfloor$ successive blocks of the source coder output bitstream into a sequence of N channel codewords of fixed length L. Suppose that we want to send successively the N packets over a memoryless noisy channel and let $\mathcal{R} = \{r_1, \ldots, r_m\}$ ($r_1 < r_2 < \cdots < r_m$) be the set of code rates corresponding to $C_1, \ldots, C_m$. Then we use an *N-packet error protection scheme* (EPS) $R = (r_{k_1}, \ldots, r_{k_N}) \in \mathcal{R}^N$, which encodes the ith information block with a channel code rate $r_{k_i} \in \mathcal{R}$. If the decoder detects an error, then the decoding is stopped, and the image is reconstructed from the correctly decoded packets. We assume that all errors can be detected.

For $i = 1, \ldots, m$, let $p(r_i)$ denote the probability of a decoding error in a packet of length L protected by code C_i. We may assume without loss of generality that $p(r_1) < \cdots < p(r_m) < 1$. For the N-packet EPS $R = (r_{k_1}, \ldots, r_{k_N})$, the probability of a decoding error in the first packet is $P_0(R) = p(r_{k_1})$, the probability that no decoding errors occur in the first i packets, $i = 1, \ldots, N-1$, with an error in the next one is $P_i(R) = p(r_{k_{i+1}}) \prod_{j=1}^{i}(1 - p(r_{k_j}))$, and the probability that all N packets are correctly decoded is $P_N(R) = \prod_{j=1}^{N}(1 - p(r_{k_j}))$. Thus the expected distortion for R is

$$E_N[d](R) = \sum_{i=0}^{N} P_i(R) d_i(R), \tag{1}$$

where $d_0(R) = d_0$ is a constant, and for $i \geq 1$, $d_i(R)$ is the reconstruction error using the first i packets. Note that if f denotes the operational distortion-rate function of the source coder, then for $i = 0, \ldots, N$, $d_i(R) = f(V_i(R))$, where $V_0(R) = 0$ and for $i \geq 1$, $V_i(R) = \sum_{j=1}^{i} v(r_{k_j})$ with $v(r_{k_j}) = \lfloor L r_{k_j} \rfloor$ being the number of source bits in the jth packet. Since the number of possible N-packet EPSs is equal to m^N, brute force cannot be used to minimize (1) when N is large. However, if we replace the minimization of (1) by the maximization of the expected number of correctly received source bits

$$E_N[r](R) = \sum_{i=0}^{N} P_i(R) V_i(R), \tag{2}$$

then an optimal solution can be computed in $O(N)$ time [3]. Maximizing (2) is reasonable for an efficient embedded coder because we expect the average distortion to decrease when the average number of correctly received source bits increases. Note, however, that the two optimizations do not necessarily yield the same EPS (see Section V). In the following, we say that an EPS that minimizes (1) is *distortion optimal* and that an EPS that maximizes (2) is *rate optimal*.

In [2], a Viterbi algorithm is used to find an approximation to a distortion-optimal EPS. However, the reported $O(N^2)$ time complexity is valid for channel code rates that are a subset of $\{\frac{p}{q}, \frac{p+1}{q}, \ldots, \frac{q-1}{q}\}$, where p and q are positive integers with $p < q$. In many important cases, including rate compatible punctured codes, the number of nodes in each stage of the Viterbi trellis does not grow linearly, and the complexity of the algorithm is exponential in the number of packets N (see Section V).

In a rate-optimal solution, the channel code rates are nondecreasing with the packet number [3]. This is not necessarily true in a distortion-optimal solution as shown by the following counter-example. Suppose that we have two packets ($N = 2$) and two channel code rates r_1 and r_2 with $r_1 < r_2$ ($m = 2$). Then $(r_1, r_1), (r_1, r_2), (r_2, r_1)$, and (r_2, r_2) are the four possible 2-packet EPSs. Suppose now that $p(r_1) = 0.09$ and $p(r_2) = 0.1$. Let $d_0 = 100, d_1(r_1, r_1) = d_1(r_1, r_2) = 95, d_1(r_2, r_1) = d_1(r_2, r_2) = 50, d_2(r_1, r_1) = 20, d_2(r_1, r_2) = d_2(r_2, r_1) = 0.001$, and $d_2(r_2, r_2) = 0.0005$. Then $E_N[d](r_2, r_1) < E_N[d](r_2, r_2) < E_N[d](r_1, r_2) < E_N[d](r_1, r_1)$. Thus, (r_2, r_1) is distortion optimal but $r_2 > r_1$. An N-packet EPS that minimizes (1) under the constraint $r_{k_1} \leq \cdots \leq r_{k_N}$ will be called *constrained distortion optimal*. In the above example, (r_2, r_2) is a constrained distortion-optimal EPS. Note that the constrained minimization reduces the number of candidates from m^N to $\binom{m+N-1}{N}$.

III. Optimal bounds

When N is large, the search space is too large to allow the determination of a distortion optimal EPS by exhaustive search. Instead of a distortion optimal solution, one could use a rate optimal one, but what is in this case the loss in quality? The following proposition shows that we are able to compute a tight upper bound on the quality loss if we assume that the operational distortion-rate function of the source coder is nonincreasing and convex.

Proposition 1: Let f be the operational distortion-rate function of the source coder. Suppose that f is nonincreasing and convex. Let T^* be a distortion-optimal N-packet EPS and let R^* be a rate-optimal N-packet EPS. Then $E_N[d](T^*) \geq f(E_N[r](R^*))$.

Proof. Let R be an N-packet EPS. Then $\sum_{i=0}^{N} P_i(R) = 1$. Thus, since f is convex, Jensen's inequality gives $E_N[d](R) \geq f(E_N[r](R))$. On the other hand, $E_N[r](R) \leq E_N[r](R^*)$. Since f is nonincreasing, this gives $f(E_N[r](R)) \geq f(E_N[r](R^*))$, which completes the proof. □

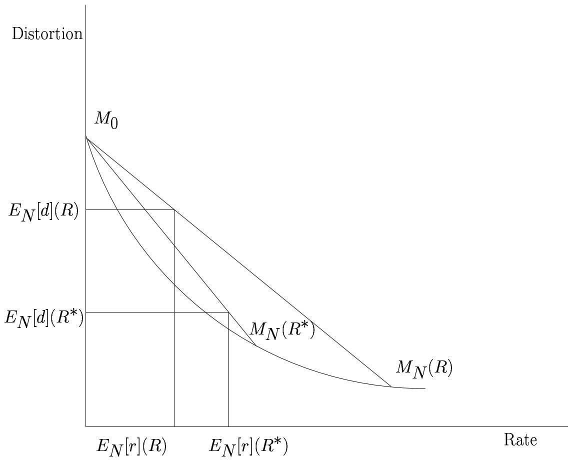

Fig. 2. Plot for Conjecture 1.

The proposition says that the approximation error $E_N[d](R^*) - E_N[d](T^*)$ is bounded by $E_N[d](R^*) - f(E_N[r](R^*))$, which can be easily computed because R^* can be determined in $O(N)$ time with the algorithm of [3].

The following conjecture [1] compares under the same assumption the total number of information bits for a rate-optimal and a distortion-optimal EPSs.

Conjecture 1: Let f be the operational distortion-rate function of the source coder. Suppose that f is nonincreasing and convex. Let T^* be a distortion-optimal N-packet EPS and let R^* be a rate-optimal N-packet EPS. Then $V_N(T^*) \leq V_N(R^*)$ and the inequality is strict if T^* is not rate optimal.

Proof. We prove the conjecture for $N = 1$. Let R be an N-packet EPS. Because $\sum_{i=0}^{N} P_i(R) = 1$, we know that $E_N[r](R) = \sum_{i=0}^{N} P_i(R)V_i(R)$ is contained in the interval $[0, V_N(R)]$. Similarly, the expected distortion $E_N[d](R) = \sum_{i=0}^{N} P_i(R)f(V_i(R))$ is contained in $[d_N(R), d_0]$. Let M_0 and $M_N(R)$ denote the points on the distortion-rate curve whose coordinates are $(0, d_0)$ and $(V_N(R), d_N(R))$, respectively. Suppose that $V_N(R) > V_N(R^*)$. Let $M_N(R^*)$ denote the point on the distortion-rate curve whose coordinates are $(V_N(R^*), d_N(R^*))$. Since $E_N[r](R) \leq E_N[r](R^*)$ and the slope of the chord $[M_0M_N(R^*)]$ is strictly smaller than the slope of the chord $[M_0M_N(R)]$, we obtain $E_N[d](R) > E_N[d](R^*)$ (see Figure 2). Thus, R cannot be distortion optimal. This proves that $V_N(T^*) \leq V_N(R^*)$. Moreover, a similar slope argument shows that if R is not rate optimal and $V_N(R) = V_N(R^*)$, then R cannot be distortion optimal, which completes the proof.

The proof works only for $N = 1$ because when $N > 1$, the barycenter of $M_0, M_1(R), \ldots, M_N(R)$ weighted with $P_0(R), P_1(R), \ldots, P_N(R)$ is not necessarily the projection of $E_N[r](R)$ on the chord $[M_0M_N(R)]$. The only thing we can say is that this point is on the segment joining the point $(E_N[r](R), f(E_N[r](R)))$ and the projection of $E_N[r](R)$ on the chord. □

The above result allows us to reduce the complexity of a distortion-based optimization by searching only in the set of EPSs with fewer information bits than a rate-optimal solution R^*. This can be done as follows.

Algorithm 1: Assume that Conjecture 1 is correct. Suppose that the distortion-rate function of the source coder is nonincreasing and convex. Then the following algorithm stops at a distortion-optimal N-packet EPS T^*.

1. Set $i = 1$. Use the algorithm of [3] to compute a rate-optimal N-packet EPS R^*. Let $A_{R^*} = \{R_1, \ldots, R_n\} \subset \mathcal{R}^N$ be the set of all N-packet EPSs R such that $V_N(R) \leq V_N(R^*)$. Suppose that the elements of A_{R^*} are ordered such that $V_N(R_1) \geq \cdots \geq V_N(R_n)$. Set $T^* = R^*$.

[1] This conjecture was erroneously given as a proposition in [10].

2. If $E_N[d](R_i) < E_N[d](T^*)$, set $T^* = R_i$.
3. If $d_N(R_i) > E_N[d](T^*)$, stop.
4. Set $i = i + 1$. If $i > n$, stop. Otherwise, go to Step 2.

Proof. The correctness of the algorithm follows from the observation that if in Step 3 the inequality $d_N(R_i) > E_N[d](T^*)$ is satisfied, then for $j \geq i$ we have $E_N[d](R_j) \geq d_N(R_j) \geq d_N(R_i) > E_N[d](T^*)$, and R_j cannot be better than T^*. □

IV. Local search algorithm

Algorithm 1 significantly accelerates the computation of a distortion-optimal EPS. However, because A_{R^*} can be huge, its computation time may be unacceptable for applications where speed is a priority. In this section, we propose a local search algorithm that rapidly finds a local minimum of (1). Experimental results in Section V show that this local minimum is near a global one. We first define the neighbors of a solution.

Definition 1: Let $\mathcal{R}$ be a set of code rates and let $R = (r_{k_1}, \ldots, r_{k_N}) \in \mathcal{R}^N$ be an N-packet EPS with nondecreasing code rates. We say that $S = (s_1, \ldots, s_N) \in \mathcal{R}^N$ is a neighbor of R if

(a) $V_N(S) < V_N(R)$.

(b) There exists a unique $i \in \{1, \ldots, N\}$ such that $s_i \neq r_{k_i}$.

(c) The code rates of S are nondecreasing.

We denote the set of neighbors of R by $\mathcal{N}(R)$ and the code rate s_i of Definition 1(b) by $r(R, S)$. We also sort the neighbors S of R by order of decreasing $r(R, S)$. More precisely, for positive integer k, the kth neighbor of an EPS R is the EPS $S \in \mathcal{N}(R)$ such that $r(R, S)$ is the kth largest code rate in the set $\{r(R, T),\ T \in \mathcal{N}(R)\}$. For example, let $\mathcal{R} = \{r_1, r_2, r_3, r_4\}$ and $R = (r_1, r_3, r_3, r_4)$ be a four-packet EPS. Then R has three neighbors, $R_1 = (r_1, r_3, r_3, r_3)$ being the first one, $R_2 = (r_1, r_2, r_3, r_4)$ the second, and $R_3 = (r_1, r_1, r_3, r_4)$ the third. Note how $r(R, R_1) = r_3 > r(R, R_2) = r_2 > r(R, R_3) = r_1$.

The local search algorithm works by iterative improvement. We start from a rate-optimal solution. Then we consider the first neighbor of this solution. If the expected distortion of this neighbor is smaller than that of the current solution, then we update the current solution and repeat the procedure; otherwise we consider the next neighbor and repeat the procedure. Note that in accordance with Conjecture 1 the neighbors of a given EPS R use fewer information bits than R. The algorithm converges after a finite number of steps because at each iteration, we decrease the total number of information bits of a solution. A pseudo-code for the local search algorithm is given below.

Local search algorithm

Initialization. 1. Set $k = 1, l = 1$, and $n = 0$. Use the algorithm of [3] to compute a rate-optimal N-packet EPS R_n.

Refinement.

2. Let r be the kth largest code rate used by R_n. Let j be the index of the first packet that R_n protects with r. If $r = r_1$, stop. Otherwise, let $r_c \in \mathcal{R}$ be the lth largest code rate smaller than r and define R_c to be the EPS obtained from R_n by protecting packet j with r_c.
3. If $E_N[d](R_c) < E_N[d](R_n)$, set $R_{n+1} = R_c, n = n + 1, k = 1, l = 1$, and go to Step 2.
4. If $j \neq 1$ and r_c is greater than the rate of packet $j - 1$ in R_n, set $l = l + 1$. If $j \neq 1$ and r_c is equal to the rate of packet $j - 1$, set $l = 1$ and $k = k + 1$. If $j = 1$ and $r_c \neq r_1$, set $l = l + 1$. If $j = 1$ and $r_c = r_1$, stop.
5. Go to Step 2.

Simulations show that the algorithm can be slightly improved by removing $k = 1$ from Step 3. We used this variant in our experimental results.

The initialization of the local search algorithm requires at most $O(Nm)$ steps [3]. In the worst case, the refinement part of the algorithm starts from the N-packet EPS $(r_m, \ldots, r_m)$ and converges to $(r_1, \ldots, r_1)$ in $O(Nm)$ steps. Thus, the time complexity of the algorithm is $O(Nm)$

in the worst case.

V. Results

In this section, we compare the time complexity and the expected mean-squared error (MSE) performance of a rate-optimal solution computed with the algorithm of [3], a solution computed with Algorithm 1 subject to the monotonicity constraint on the channel code rates, the solution computed with the Viterbi algorithm of [2] (this algorithm also uses the monotonicity constraint on the channel code rates), and the solution of the local search algorithm of Section IV. The CPU time was measured on an MIPS R12000 processor of an SGI Origin 200 with a main memory size of 1536 Megabytes. The test images were the standard 8 bits per pixel (bpp) gray scale 512×512 Lenna, Goldhill, and Peppers. The embedded source coders were Jim Fowler's implementation of the SPIHT algorithm [11] and the Kakadu implementation of JPEG2000 in the distortion scalable mode [9]. Note that the operational distortion-rate functions of these coders can be well modeled with nonincreasing convex functions [12]. The packets were sent over a binary symmetric channel (BSC) with bit error rate (BER) 0.1. We recall that N and L denote the number of packets sent and the length of a channel codeword, respectively. Thus, the transmission rate in bpp is $R_T = NL/n^2$ for $n \times n$ images.

In a first experiment, the embedded bitstream was protected with a concatenation of a CRC-32 coder and a rate-compatible punctured turbo (RCPT) coder [13]. The generator polynomial of the CRC code was (32,26,23,22,16,12,11,10,8,7,5,4,2,1,0). The turbo coder consisted of two identical recursive systematic convolutional coders with memory length four and generator polynomials $(31, 27)$ (octal). The mother code rate was $20/60 = 1/3$, and the puncturing rate was 20, yielding 41 possible channel code rates. The length of a packet was equal to $L = 2048$ bits, consisting of a variable number of source bits, 32 CRC bits, 4 bits to set the turbo encoder into a state of all zeroes, and protection bits. We used iterative maximum a posteriori decoding, which was stopped if no correct sequence was found after 20 iterations.

The probability of a packet decoding error for each code rate was computed with simulations. We used only a subset of the set of 41 admissible RCPT code rates. Indeed, when many code rates have the same decoding error probability, only the largest one has to be kept. Also one can ignore any code rate whose residual bit error rate is greater than the BER. For BER 0.1, the set of retained code rates was $r_1 = \frac{20}{58}, r_2 = \frac{20}{56}, r_3 = \frac{20}{52}, r_4 = \frac{20}{50}$, and $r_5 = \frac{20}{48}$. The corresponding numbers of source bits per packet were $v(r_1) = 670, v(r_2) = 695, v(r_3) = 751, v(r_4) = 783$, and $v(r_5) = 817$. The probabilities of packet decoding failure were $p(r_1) = 0, p(r_2) = 0.00001, p(r_3) = 0.0002, p(r_4) = 0.00117$, and $p(r_5) = 0.00449$.

Table I compares the performance of the algorithms for the SPIHT bitstream of the Lenna image. The bound of Proposition 1 is also provided. We point out that the transmission rate does not include the side information needed to specify the solution when a distortion-based error protection is used. Since we consider only EPSs with nondecreasing code rates and since generally $m < N$, one can use run-length encoding to compress the overhead to $m\lceil \log_2 N \rceil + (m-1)\lceil \log_2 m \rceil$ bits in the worst case. Table II and III show the results of similar experiments for Goldhill and Peppers, respectively.

As expected, the best MSE results were obtained with Algorithm 1. Since these results are close to the lower bound, we conclude that they are near-optimal. Thus, the monotonicity constraint on the code rates does not seem to be a harmful restriction. The fastest algorithm was the one of [3]; and except for the lowest transmission rates, it provided high-quality solutions. The local search algorithm was able to improve the solutions of [3]. Moreover, its performance was comparable to that of Algorithm 1, while its time complexity was much lower than that of the Viterbi algorithm. For example, at transmission rate 1 bpp, we have $N = 128$ packets. Thus, the total number of candidates is 5^{128}, and the number of candidates with nondecreasing code rates is 12,082,785. For the Lenna image, the local search algorithm checked only 51 solutions in the refinement stage. In contrast, the total number of considered paths in the Viterbi trellis

Total rate (bpp)	Bound MSE	Alg. 1 MSE	Alg. 1 Time (s)	RO MSE	RO Time (s)	LS MSE	LS Time (s)	Viterbi MSE	Viterbi Time (s)
0.25	68.94	70.42	10.72	74.90	< 0.01	70.52	0.04	70.49	24.87
0.5	35.38	36.26	549.32	37.28	< 0.01	36.28	0.08	36.38	243.85
0.75	23.78	24.39	4645.71	25.34	< 0.01	24.42	0.12	24.56	934.86
1.0	17.48	18	21424.59	19.20	< 0.01	18.01	0.22	18.02	2489.11

TABLE I

CPU time in seconds and expected MSE at various transmission rates for Algorithm 1, a rate-optimal solution (RO), a solution found by the local search algorithm (LS), and one obtained with the Viterbi algorithm [3]. Results are for the 512×512 Lenna image, the SPIHT source coder, an RCPT channel coder, and a BSC with BER = 0.1.

Total rate (bpp)	Bound MSE	Alg 1 MSE	Alg 1 Time (s)	RO MSE	RO Time (s)	LS MSE	LS Time (s)	Viterbi MSE	Viterbi Time (s)
0.25	102.37	103.67	9.42	108.24	< 0.01	103.69	0.04	103.72	23.68
0.5	67.50	68.28	383.49	69.07	< 0.01	68.29	0.07	68.46	238.70
0.75	50.28	51.01	3775.22	51.86	< 0.01	51.09	0.1	51.36	914.15
1.0	40.56	41.24	19073.03	42.23	< 0.01	41.27	0.16	41.37	2462.9

TABLE II

CPU time in seconds and expected MSE at various transmission rates for Algorithm 1, a rate-optimal solution (RO), a solution found by the local search algorithm (LS), and one obtained with the Viterbi algorithm [3]. Results are for the 512×512 Goldhill image, the SPIHT source coder, an RCPT channel coder, and a BSC with BER = 0.1.

was equal to 1,359,439, of which 20,831 had the maximum length of 128.

The Viterbi algorithm of [2] is much faster for channel code rates in $\{\frac{p}{q}, \frac{p+1}{q}, \ldots, \frac{q-1}{q}\}$, where $0 < p < q$. We now compare the local search algorithm with the Viterbi algorithm for such channel code rates. Let us consider, for example, the punctured turbo codes used in [2]. The code rates are 11/12, 10/12, 9/12, 8/12, 6/12, 5/12, and 4/12. For BER = 0.1, only code rates 4/12, 5/12, and 6/12 with respective packet decoding error probabilities 0.00001, 0.0003, and 0.88 were used. Indeed, the probability of decoding error of the next code rate was greater than the BER. Table IV shows that even with the settings of [2], the local search algorithm was much faster than the Viterbi algorithm, whereas the MSE-performance of the two algorithms was almost identical.

VI. Conclusion

For the original system of Sherwood and Zeger [1], a distortion-optimal EPS can be computed with dynamic programming in $O(R^2)$ time where R is the transmission rate [5]. However, in the

Total rate (bpp)	Bound MSE	Alg. 1 MSE	Alg. 1 Time (s)	RO MSE	RO Time (s)	LS MSE	LS Time (s)	Viterbi MSE	Viterbi Time (s)
0.25	66.86	68.75	11.67	76.64	< 0.01	68.78	0.04	68.79	23.51
0.5	35.88	36.78	555.97	38.36	< 0.01	36.81	0.08	36.87	234.96
0.75	26.13	26.71	4811.72	28.33	< 0.01	26.74	0.12	26.78	903.58
1.0	20.67	21.15	21844.19	22.93	< 0.01	21.16	0.21	21.18	2448.7

TABLE III

CPU time in seconds and expected MSE at various transmission rates for Algorithm 1, a rate-optimal solution (RO), a solution found by the local search algorithm (LS), and one obtained with the Viterbi algorithm [3]. Results are for the 512×512 Peppers image, the SPIHT source coder, an RCPT channel coder, and a BSC with BER = 0.1.

Total rate (bpp)	Bound MSE	Alg. 1 MSE	Alg. 1 Time (s)	RO MSE	RO Time (s)	LS MSE	LS Time (s)	Viterbi MSE	Viterbi Time (s)
0.25	65.68	66.01	0.01	66.91	< 0.01	66.01	0.01	66.01	0.02
0.5	31.53	32.16	0.02	32.68	< 0.01	32.16	0.01	32.16	0.14
0.75	20.41	21.14	0.08	21.78	< 0.01	21.16	0.01	21.14	0.53
1.0	15.30	15.92	0.22	16.78	< 0.01	15.92	0.02	15.92	1.47

TABLE IV

Same experiments as in Table I for the 512×512 Lenna image, JPEG2000, the channel coder used in [2], packet length 512 bits, and a BSC with BER = 0.1.

fixed-length channel codeword setting, there is no algorithm that computes a distortion-optimal EPS in reasonable time. The best previous approximate solution is based on a Viterbi algorithm [2]. We provided an easily computable tight lower bound on the lowest expected distortion of a joint source-channel coding system. This lower bound is useful to evaluate the quality of approximate solutions. We also proposed a local search algorithm that finds a high-quality local minimum of the expected distortion. Our goal was to minimize the expected distortion. If we prefer instead to maximize the expected PSNR, then the same techniques can be used under the assumption that the operational PSNR-rate function is nondecreasing and concave.

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